Measuring risk in fixed income portfolios using yield curve models

João F. Caldeira
Departmento de Economia
Universidade Federal do Rio Grande do Sul

Guilherme V. Moura
Departmento de Economia
Universidade Federal de Santa Catarina

André A. P. Santos*
Departmento de Economia
Universidade Federal de Santa Catarina

June 3, 2013

Abstract

We propose a novel approach to measure risk in fixed income portfolios in terms of value-at-risk (VaR). We use closed-form expressions for the vector of expected bond returns and for the covariance matrix of bond returns based on a general class of well established term structure factor models, including the dynamic versions of the Nelson-Siegel and Svensson models, to compute the parametric VaR of a portfolio composed of fixed income securities. The proposed approach is very flexible as it can accommodate alternative specifications to model the yield curve and also alternative specifications to model the conditional heteroskedasticity in bond returns. An empirical application involving a data set with 15 fixed income securities with different maturities indicate that the proposed approach delivers very accurate VaR estimates.

JEL: C53; E43; G17.
key-words: backtesting; dynamic conditional correlation (DCC); forecast; maximum likelihood; value-at-risk

*Universidade Federal de Santa Catarina, Departamento de Economia. Campus Universitário Trindade 88049-970, Florianopolis, SC - Brasil. E-mail: andre.portela@ufsc.br.

1 Introduction

Value-at-risk (VaR) is now established as one of the most important risk measures designed to control and to manage market, which is the risk of losses on positions in equities, interest rate related instruments, currencies and commodities due to adverse movements in market prices, and to determine the amount of capital subject to regulatory control Berkowitz & O'Brien [2002], Santos *et al.* [2012b]. Moreover, the Basel Accords also establish penalties for inadequate models and, consequently, create incentives to pursue accurate VaR estimates. Therefore, VaR is a widely used measure of market risk which has become one of the most important issues in risk management, [see e.g. Jorion, 2006]. VaR may be defined as the worst scenario that is expected to occur with a large probability for a portfolio given by a linear combination of the returns of the multivariate series. In fact, value at risk measures the worst case loss (i.e. a threshold loss) at the given confidence level and investment horizon conditions.

Not least because of the Basel accords, but also because of its popularity in the industry, VaR has attracted a considerable amount of theoretical and applied research. A large number of recent studies devoted attention to finding what is the most appropriate approach to model and forecast the VaR of a portfolio of assets. This is generally conducted by backtesting a set of alternative specifications and checking whether the number of VaR violations (i.e., instances in which the actual portfolio loss exceeds the estimated VaR) is adequate; see, for instance, Christoffersen *et al.* [2001], Berkowitz & O'Brien [2002], Brooks & Persand [2003], Giot & Laurent [2003], Giot & Laurent [2004], Bauwens *et al.* [2006], Christoffersen [2009] and McAleer [2009], among many others. These studies, in general, focus on two main approaches to obtain the portfolio VaR, which are either to use a multivariate model for the system of individual asset returns, or a univariate specification to model directly the portfolio returns. More recently, Santos *et al.* [2012a] compared both approaches and conclude that the multivariate approach provides more accurate VaR estimates.

The vast majority of the existing evidence on VaR modeling focus on measuring the risk of equity portfolios Giot & Laurent [2003], Engle & Manganelli [2004], Giot & Laurent [2004], Galeano & Ausin [2010]. Surprisingly, the literature on VaR modeling of fixed income securities is very thin. This seems to be an important gap in the literature, since fixed income securities play a fundamental role in the composition of diversified portfolios held by institutional investors. One explanation for the relative lack of literature on VaR modeling for bond portfolios is the relative stability and low historical volatility of this asset class, which discouraged the use of sophisticated methods to measure and to manage the market risk of interest rate related instruments. However, this situation has been changing dramatically in recent years, even in markets where these assets have low default probability Korn & Koziol [2006]. The recurrence of turbulent episodes in global markets usually brings high volatility to bond prices, which increases the importance of adopting appropriate techniques for measuring the risk of bond portfolios.

Under a multivariate setting in which a portfolio of assets is concerned, the VaR computation usually requires two main ingredients, namely the vector of expected returns and their covariance matrix. For instance, Ferreira [2005] models the expected returns of the French and German short rates using an autoregressive specification and alternative specifications to model the variances and covariances of the two rates. Ferreira & Lopez [2005] consider the problem of modeling and forecasting the VaR of an equally-weighted portfolio of short-term fixed-income positions in the U.S. dollar, German deutschemark, and Japanese yen using alternative range of multivariate volatility specifications. Alternatively, Vlaar [2000] considers the VaR computation of Dutch fixed-interest securities with different maturities using historical simulation, Monte Carlo simulation methods and a set of univariate volatility models, including univariate models with alternative distribution assumptions.

In this paper, we amend the literature on VaR-based risk measurement by putting forward a novel approach to measure risk in bond portfolios. Our approach significantly differs from the existing ones as it is built upon a general class of well established term structure factor models such as the dynamic version of the Nelson-Siegel model proposed by Diebold & Li [2006], and the four factor version proposed by Svensson [1994]. These models have been successfully employed in forecasting yields; see, for instance, De Pooter [2007], Diebold & Rudebusch [2011], Caldeira *et al.* [2010b], and Rezende & Ferreira [2011] for an analysis of the predictive performance of factor models for the term structure. Moreover, since it is based on factor specifications, our approach is parsimonious and suitable for high-dimensional applications in which a large number of fixed income securities is involved. Finally, we take a step further with respect to the existing evidence and obtain closed-form expressions for the vector expected bond returns and for the covariance matrix of bond returns based on yield curve models to compute the VaR of a bond portfolio. We show that the proposed approach is very flexible as it can accommodate a wide range of alternative specifications to model the yield curve and also alternative specifications to model the conditional heteroskedasticity in bond returns.

We provide empirical evidence of the applicability of the proposed approach by considering a data set similar to the one used by Almeida & Vicente [2009] which is composed of constant-maturity future contracts of the Brazilian Inter Bank Deposit Future Contract (DI-futuro) which is equivalent to a zero-coupon bond and is highly liquid (293 million contracts worth US$ 15 billion traded in 2010). The market for DI-futuro contracts is one most liquid interest rate market in the world. Many banks, insurance companies, and investors trade a large number of DI-futuro contracts as an investment and hedging instrument, or to exploit an arbitrage opportunity. While DI-futuro contracts are riskless if held to maturity, they have risk exposure from DI-futuro yield changes during the investment horizon. Hence, the DI-futuro yield curve movements play an important role for risk managers. Many institutions and specialized vendors such as Bloomberg, offers the Brazilian yield curve data online for fixed income investors. The data set considered in the paper contains DI-futuro contracts traded on the Brazilian Mercantile and Futures Exchange (BM&F) with fixed maturities of 1, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 42 and 48 months. In order to obtain out-of-sample VaR estimates for the 1%, 2.5%, and 5% levels for the equally-weighted portfolio, we use the dynamic versions of the Nelson-Siegel and Svensson yield curve models to derive estimates for the vector of expected bond returns and alternative GARCH-type specifications to model the covariance matrix of bond returns. A total of 16 alternative specifications is proposed and their performance evaluated by means of a backtesting analysis based on independence, unconditional coverage and conditional coverage tests of Christoffersen [1998]. These tests, though appropriate to evaluate the accuracy of a single specification, can provide ambiguous decision about which candidate model is better; see Santos *et al.* [2012a] for a discussion. Therefore, we enhance the traditional backtesting analysis by using statistical tests designed to compare the predictive performance among several candidate models applying the comparative predictive ability (CPA) test proposed by Giacomini & White [2006]. Our results indicate that the proposed approach is able to deliver accurate VaR estimates for all VaR levels considered in the paper. In particular, we find the VaR estimates obtained with the Nelson-Siegel model with factor dynamics given by a vector autoregressive specification, and conditional covariance matrix given by a dynamic conditional correlation (DCC-GARCH) model, to be the most accurate among all specifications considered and for three VaR levels considered in the paper.

The paper is organized as follows. Section 2 describes the factor models used for modeling the term structure, as well as the econometric specification for the conditional heteroscedasticity of the factors, and provides closed-form expression for the first two conditional moments. Section 2.4 presents the methodology for VaR computation whereas 3 discusses the estimation strategy. Section 4 presents an empirical applications. Finally, Section 5 brings concluding remarks.

2 Value-at-risk using yield curve models

In this section we consider the use of dynamic factor models for the yield curve to obtain VaR estimates. Factor models for the term structure of interest rates allow us to obtain closed form expressions for the expected yields, as well as for their conditional covariance matrix. From these moments, we show how to compute the distribution of bond prices and bond returns, which will later be used as an input to compute the VaR of a portfolio of bonds.

2.1 Dynamic factor models for the yield curve

We consider a set of time series of bond yields with N different maturities, $\tau_1, \ldots, \tau_N$. The yield at time t of a security with maturity τ_i is denoted by $y_t(\tau_i)$ for $t = 1, \ldots, T$. The $N \times 1$ vector of all yields at time t is given by

$$y_t(\tau) = \left(y_t(\tau_1) \ldots y_t(\tau_N)\right), \quad t = 1, \ldots, T.$$

The general specification of the dynamic factor model is given by

$$y_t = \Lambda(\lambda) f_t + \varepsilon_t, \quad \varepsilon_t \sim \text{NID}\left(0, \Sigma_t\right), \quad t = 1, \ldots, T, \tag{1}$$

where $\Lambda(\lambda)$ is the $N \times K$ matrix of factor loadings, f_t is a $K-$dimensional stochastic process, ε_t is the $N \times 1$ vector of disturbances and Σ_t is an $N \times N$ conditional covariance matrix of the disturbances. As usual in the yield curve literature, we restrict the covariance matrix Σ_t to be diagonal [see Diebold *et al.* , 2006]. This means that the covariance between the yields with different maturities is explained solely by the common latent factor f_t. The dynamic factors f_t are modeled by the following stochastic process

$$f_t = \mu + \Upsilon f_{t-1} + \eta_t, \quad \eta_t \sim \mathrm{NID}\left(0, \Omega_t\right), \quad t = 1, \ldots, T, \tag{2}$$

where μ is a $K \times 1$ vector of constants, Υ is the $K \times K$ transition matrix, and Ω_t is the conditional covariance matrix of the disturbance vector η_t, which is independent of the vector of residuals ε_t $\forall t$.

The specification for f_t is general. Therefore, it is possible to model its dynamics using a variety of process [see Jungbacker & Koopman, 2008]. In modeling yield curves the usual specification for f_t is a vector autoregressive process of lag order 1 [Diebold *et al.* , 2006, Caldeira *et al.* , 2010b].

Next we present the factor models for the yield curve considered in this paper. The specifications considered are the two main variants of the original formulation of the Nelson & Siegel [1987] factor model, namely the dynamic Nelson-Siegel model proposed by Diebold & Li [2006], and the 4-factor extension proposed by Svensson [1994]. The alternative Nelson-Siegel specifications considered are all nested and can therefore be captured in the general formulation in (1) and (2) with different restrictions imposed on the loading matrix $\Lambda(\lambda)$.

2.1.1 Dynamic Nelson-Siegel model

The model of Nelson & Siegel [1987] and its extension by Svensson [1994] are widely used by central banks and other market participants as a model for the term structure of interest rates [BIS, 2005, Gimeno & Nave, 2009]. Academic studies have provided evidence that the model can also be a valuable tool for forecasting the term structure, see for instance Diebold & Li [2006]. We look into the two main variants of the model, namely the original formulation of Nelson & Siegel [1987] and its extension by Svensson [1994]. Diebold & Li [2006] proposed a dynamic version of the Nelson & Siegel [1987] exponential components framework. They obtained very good results in terms of out of sample yield curve forecasting. The starting point is the following model for the continuously compounded spot rate, with $\tau = T - t$:

$$y_t(\tau) = \beta_1 + \beta_2 \left(\frac{1 - e_{-\lambda\tau}}{\lambda\tau} \right) + \beta_3 \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{\lambda\tau} \right), \tag{3}$$

Equation (3) is the "original" Nelson-Siefel yield curve specification, where $f_t = (\beta_1, \beta_2, \beta_3)$ and λ are parameters. Diebold & Li [2006] interpreted equation (4) in a dynamic fashion as a latent factor model in which $\{\beta_1, \beta_2, \beta_3\}$ are time-varying level, slope, and curvature factors, and the terms that multiply these factors are factor loadings:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e_{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{\lambda\tau}\right). \tag{4}$$

with $y_t(\tau)$ denoting the continuously-compounded zero-coupon nominal yield at maturity τ, and $\beta_{1t}, \beta_{2t}, \beta_{3t}$ and λ_t are (time-varying) parameters. Equation (4) is a dynamic version as parameters are allowed to vary through time. Therefore the model is typically known as the Dynamic Nelson-Siegel model (DNS).

2.1.2 The Svensson model

A number of authors have proposed extensions to the Nelson-Siegel model that enhance flexibility. For example, Svensson [1994] included another exponential term, similar to the third, but with a different decaying parameter. The Svensson dynamic four factor model is written as:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_1\tau}}{\lambda_1\tau} - e^{-\lambda_1\tau}\right) + \beta_{4t}\left(\frac{1-e^{-\lambda_2\tau}}{\lambda_2\tau} - e^{-\lambda_2\tau}\right). \tag{5}$$

The fourth component can be interpreted as a second curvature. The Svensson model can fit term structure shapes with more than one local maximum or minimum along the maturity spectrum.

2.2 Conditional covariance of the factor models for the yield curve

Forecasting volatility of interest rates remains an important challenge in financial econometrics.[1] A rich body of literature has shown that the volatility of the yield curve is, at least to some extent, related to the shape of the yield curve. For instance, the volatility of interest rates is usually high when interest rates are high and when the yield curve exhibits more curvature (see Cox *et al.* [1985], Litterman *et al.* [1991], and Longstaff & Schwartz [1992], among others). This suggests that the shape of the yield curve is a potentially useful instrument for forecasting volatility.

Despite the large amount of studies dealing with fitting and forecasting of the yield curve, only recently attention has been turned to the presence of conditional heteroskedasticity in the term structure of interest rates.[2] In most cases, the models for the yield curve adopt the assumption of constant volatility for all maturities. This issue is particularly important since the assumption of constant interest rate volatility has important practical implications for risk management policies, as it neglects the time-varying characteristic of interest rate risk. Furthermore, interest rate hedging and arbitrage operations are also influenced by the presence of time-varying volatility as, in these operations, it is often necessary to compensate for the market price of interest rate risk. Another important implication is that in the presence of conditional volatilities the confidence intervals for the forecasts obtained from these models will be possibly miscalculated in finite samples. Some

[1] See Poon & Granger [2003] and Andersen & Benzoni [2010] for recent surveys on volatility forecasting.

[2] See, for instance, Filipovic [2009], for a review on interest rate modeling.

recent approaches designed to overcome these limitations have been proposed proposed by Bianchi *et al.* [2009], Haustsch & Ou [2012], Koopman *et al.* [2010] and Caldeira *et al.* [2010a].

In this paper, the effects of time-varying volatility are incorporated using a multivariate GARCH specification proposed by Santos & Moura [2012]. To model Ω_t, the conditional covariance matrix of the factors in (2), alternative specifications can be considered, including not only multivariate GARCH models but also multivariate stochastic volatility models [see Harvey *et al.* , 1994, Aguilar & West, 2000, Chib *et al.* , 2009]. In this paper, we consider the dynamic conditional correlation model (DCC) proposed by Engle [2002], which is given by:

$$\Omega_t = D_t \Psi_t D_t, \tag{6}$$

where D_t is a $K \times K$ diagonal matrix with diagonal elements given by $h_{f_{kt}}$, where $h_{f_{kt}}$ is the conditional variance of the k-th factor, and Ψ_t is a symmetric correlation matrix with elements $\rho_{ij,t}$, where $\rho_{ii,t} = 1$, $i, j = 1, \ldots, K$. In the DCC model, the conditional correlation $\rho_{ij,t}$ is given by:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} q_{jj,t}}}, \tag{7}$$

where $q_{ij,t}$, $i, j = 1, \ldots, K$, are the elements of the $K \times K$ matrix Q_t, which follows a GARCH-type dynamics:

$$Q_t = (1 - \alpha - \beta)\, \bar{Q} + \alpha z_{t-1} z'_{t-1} + \beta Q_{t-1}, \tag{8}$$

where $z_{f_t} = (z_{f_{1t}}, \ldots, z_{f_{kt}})$ is the standardized vector of returns of the factors, whose elements are $z_{f_{it}} = f_{it}/\sqrt{h_{f_{it}}}$, $\bar{Q}$ is the unconditional covariance matrix z_t, α e β are non negative scalar parameters satisfying $\alpha + \beta < 1$.

To model the conditional variance of the measurement errors ε_t in (1), we assume that Σ_t is a diagonal matrix with diagonal elements given by $h_{t\varepsilon_i}$, where $h_{t\varepsilon_i}$ is the conditional variance of ε_i. Moreover, a procedure similar to Cappiello *et al.* [2006] is applied and alternative specifications of the univariate GARCH type are used to model $h_{t\varepsilon_i}$. In particular, we consider the GARCH model of Bollerslev [1986], the asymmetric GJR-GARCH model Glosten *et al.* [1993], the exponential GARCH (EGARCH) model of Nelson [1991], the threshold GARCH (TGARCH) model of Zakoian [1994], the asymmetric exponent GARCH (APARCH) model of Ding *et al.* [1993], asymmetric GARCH (AGARCH) model of Engle [1990], and the non-linear asymmetric GARCH (NAGARCH) model of Engle & Ng [1993]. In all models, without loss of generality, their simplest form is adopted in which the conditional variance depends on one lag of both past returns and conditional variances. The Appendix lists the exact specifications of each of these models. The same procedure is applied to the choice of the GARCH specification for the conditional variance of the factors in (6). In all cases, the choice of the specification used is based on Akaike Information Criterion (AIC).

2.3 Expected bond returns and the conditional covariance matrix of bond returns

As we discuss in Section 2.4, the computation of the VaR requires estimates of the expected return of each bond, as well as the covariance matrix of the set of bond returns in the portfolio. However, the factor models for the term structure of interest rates discussed above are designed to model only bond *yields*. Nevertheless, it is possible to obtain expressions for the expected bond return and for the conditional covariance matrix of bond returns based on the distribution of the expected yields. The following proposition defines this distribution.

Proposition 1. *Given the system of equations in (1) and (2), the distribution of expected yields $y_{t|t-1}$ is $N(\mu_{y,t}, \Sigma_{y,t})$ with $\mu_{y,t} = \Lambda f_{t|t-1}$ and $\Sigma_{y,t} = \Lambda\Omega_{t|t-1}\Lambda' + \Sigma_{t|t-1}$, where $f_{t|t-1}$ is a one-step-ahead forecast of the factors and $\Sigma_{t|t-1}$ and $\Omega_{t|t-1}$ are one-step-ahead forecasts of the conditional covariance matrices in (1) and (2), respectively.*

Proof. Taking expectation of the factor model for the yields in (1), we have

$$\mu_t = E_{t-1}\left[y_t\right] = \mathbf{\Lambda}\left(\lambda\right) E_{t-1}\left[f_t\right] = \mathbf{\Lambda}\left(\lambda\right) f_{t|t-1} \tag{9}$$

where $f_{t|t-1}$ are one-step-ahead predictions of the factors. The corresponding conditional covariance matrix is given by:

$$\begin{aligned}
\Sigma_{y_t} &= E_{t-1}\left[\left(y_t - E_{t-1}\left[y_t\right]\right)\left(y_t - E_{t-1}\left[y_t\right]\right)\right] \\
&= E_{t-1}\left[\left(\Lambda f_t + \varepsilon_t - \Lambda E_{t-1}\left[f_t\right]\right)\left(\Lambda f_t + \varepsilon_t - \Lambda E_{t-1}\left[f_t\right]\right)'\right] \\
&= E_{t-1}\left[\left(\Lambda\left(f_t - E_{t-1}\left[f_t\right]\right) + \varepsilon_t\right)\left(\Lambda\left(f_t - E_{t-1}\left[f_t\right]\right) + \varepsilon_t\right)'\right] \\
&= E_{t-1}\left[\left(\Lambda\left(\mu + \Upsilon f_{t-1} + \eta_t - \mu - \Upsilon f_{t-1}\right) + \varepsilon_t\right)\left(\Lambda\left(\mu + \Upsilon f_{t-1} + \eta_t - \mu - \Upsilon f_{t-1}\right) + \varepsilon_t\right)'\right] \\
&= E_{t-1}\left[\left(\Lambda\eta_t + \varepsilon_t\right)\left(\Lambda\eta_t + \varepsilon_t\right)'\right] \\
&= E_{t-1}\left[\Lambda\eta_t\eta_t'\Lambda' + \varepsilon_t\varepsilon_t'\right] \\
&= \Lambda E_{t-1}\left[\eta_t\eta_t'\right]\Lambda' + E_{t-1}\left[\varepsilon_t\varepsilon_t'\right] \\
&= \Lambda\Omega_{t|t-1}\Lambda' + \Sigma_{t|t-1}.
\end{aligned}$$

since cross-product between η_t and ε_t vanishes because of independence. □

Using the results of Proposition 1, we next show that it is possible to derive the distribution of expected fixed-maturity bond prices. Taking into account that the price of a bond at time t, $P_t(\tau)$, is the present value at time t of \$1 receivable τ periods ahead, and letting $y_{t|t-1}$ denote the one step ahead forecast of its continuously compounded zero-coupon nominal yield to maturity, we obtain the vector of expected bond prices $P_{t|t-1}$ for all maturities:

$$P_{t|t-1} = \exp\left(-\tau \otimes y_{t|t-1}\right), \tag{10}$$

where $\otimes$ is the Hadamard (elementwise) multiplication and τ is the vector of maturities. Since $y_{t|t-1}$ follows a Normal distribution, $P_{t|t-1}$ has a log-normal distribution.

Note that the log-returns can be written as

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = \log P_t - \log P_{t-1} = -\tau \otimes (y_t - y_{t-1}) \,. \tag{11}$$

It is possible to find a closed form expression for the vector of expected returns of bonds as well as for their conditional covariance matrix using (11). Proposition 2 defines these expressions.

Proposition 2. *Given the system of equations in (1) and (2) and Proposition 1, the vector of expected log-returns for bonds, $\mu_{r_{t|t-1}}$, and their conditional covariance matrix $\Sigma_{r_{t|t-1}}$, which is positive-definite $\forall t$, are given by:*

$$\mu_{r_{t|t-1}} = -\tau \otimes \mu_{y,t} + \tau \otimes y_{t-1}, \tag{12}$$

$$\Sigma_{r_{t|t-1}} = \tau'\tau \otimes \left[\underbrace{\Lambda\Omega_{t|t-1}\Lambda' + \Sigma_{t|t-1}}_{\Sigma_{y,t}}\right]. \tag{13}$$

Proof. Using the log-return expression, we get:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = \log P_t - \log P_{t-1} = -\tau \otimes (y_t - y_{t-1}) \,. \tag{14}$$

Since $y_{t|t-1} \sim N(\mu_t, \Sigma_{y_t})$ where μ_t e Σ_{y_t} are defined in Proposition 1, it is known that the expected returns $r_{t|t-1}$ follow $N(\mu_{r_t}, \Sigma_{r_t})$ where

$$\mu_{r_{t|t-1}} = -\tau \otimes (E_{t-1}[y_t] - E_{t-1}[y_{t-1}]) = -\tau \otimes \mu_t + \tau \otimes y_{t-1}, \tag{15}$$

$$\Sigma_{r_{t|t-1}} = \tau'\tau \otimes \left[\underbrace{\Lambda\Omega_{t|t-1}\Lambda' + \Sigma_{t|t-1}}_{\Sigma_{y_t}}\right]. \tag{16}$$

The positivity of the matrix Σ_{r_t} can be demonstrated as follows. The first term in brackets, $\Lambda\Omega_t\Lambda'$, is positive-definite since $\Omega_{t|t-1}$ is diagonal and possesses only positive elements on its diagonal. The second term, $\Sigma_{t|t-1}$ is positive-definite for the same reason. Since τ contains only positive elements, $\tau'\tau$ is also a positive-definite matrix. Finally, Schur product theorem ensures that the Hadamard product between Σ_{y_t} and $\tau'\tau$ is positive definite. □

The results in Proposition 2 show that it is possible to obtain closed form expressions for the expected bond log-returns and their covariance matrix based on yield curve models such as the ones by Nelson & Siegel [1987] and Svensson [1994]. These estimates are key ingredients to the problem of portfolio selection based on mean-variance paradigm proposed by Markowitz, as discussed in section 4 bellow.

As pointed out by Litterman & Scheinkman [1991], the return on a fixed maturity zero-coupon bond can be decomposed into two parts. The first part is a result of the capitalization received due to ageing of the bond and the second part is attributed to the change in market prices of constant maturity bonds. Furthermore, Litterman & Scheinkman [1991] point out that the first part is deterministic, while the second part is subject to uncertainty regarding the changes in prices. Clearly, portfolio optimization is only concerned with the second part.

However, for comparison with other benchmarks, it is also necessary to compute the deterministic part of the return. The total return will be given by the income generated by the capitalization based on the interest rate on the bond, plus capital appreciation given by the variation in market prices. Following Jones *et al.* [1998] and de Goeij & Marquering [2006], the total return (between t and $t+h$) on a bond with fixed maturity τ is given by:

$$R_{t+h}(\tau) = \frac{P_t(\tau)}{P_{t-h}(\tau)} - 1 + \frac{h}{252} y_{t-h}(\tau) = \exp(r_{y,t+h}) - 1 + \frac{h}{252} y_{t-h}, \tag{17}$$

where h is given on weekdays and $r_{y,t+h}$ is the log-return generated by changes in yields of fixed maturities from period t to $t+h$.

2.4 VaR computation

We now consider the computation of the VaR for bond portfolios using the yield curve models discussed before. As we show next, the closed form expressions for the vector of bond portfolio returns and their covariance matrix discussed in Section 2.3 can be applied to the computation of the bond portfolio VaR in a straightforward way. Throughout the paper, we focus on the portfolio VaR for a long position in which traders have bought fixed income securities and wish to measure the risk associated to a decrease in their market prices. In this sense, we are interested in measuring the risk associated to increases in bond yields, which is related to decreasing prices, and thus to negative returns. Moreover, we consider an equally-weighted portfolio, which has been extensively used in the empirical literature; see, for instance, Zaffaroni [2007], DeMiguel *et al.* [2009], and Santos *et al.* [2012a].

Denote by $R_{t+h} = (r_{1,t+h}, \ldots, r_{N,t+h})'$ the vector of h-period returns (between t and $t+h$) of the N bonds included in the portfolio. The bond portfolio return is given by $r_{p,t+h} = w_t' R_{t+h}$, where w_t is the vector of portfolio weights to be determined at time t. The portfolio VaR at time t for a given holding period h and confidence level ϑ is given by the ϑ-quantile of the conditional distribution of the bond portfolio return. Thus, $\mathrm{VaR}_t(h,\vartheta) = F^{-1}_{p,t+h}(\vartheta)$, where $F^{-1}_{p,t+h}$ is the inverse of the cumulative distribution function of $r_{p,t+h}$. Throughout the paper we focus on the portfolio VaR for a holding period of $h = 1$ day at $\vartheta = 1\%$, $\vartheta = 2.5\%$, and $\vartheta = 5\%$, which are the most common risk levels used to compute the VaR. Therefore, from now on, we omit the arguments h

and ϑ from the definition of the VaR.

When the distribution of bond log-returns is expressed in terms of its two first conditional moments, the portfolio return can be represented as

$$r_{p,t+1} = \mu_{p,t+1} + \sigma_{p,t+1} z_{p,t+1}, \tag{18}$$

where the standardized unexpected returns $z_{p,t+1}$ are independent and identically distributed with mean equal to zero and unit variance. $\mu_{p,t+1}$ and $\sigma_{p,t+1}$ are the conditional mean and standard deviation of the bond portfolio return, given by

$$\mu_{p,t+1} = w_t' \mu_{r_{t+1}} \tag{19}$$

and

$$\sigma^2_{p,t+1} = w_t' \Sigma_{r_{t+1}} w_t, \tag{20}$$

where $\mu_{r_{t+1}}$ is the $N \times 1$ vector of conditional mean returns for the N individual assets and $\Sigma_{r_{t+1}}$ is their $N \times N$ conditional covariance matrix defined in (12) and (13), respectively. The portfolio VaR is then given by

$$\text{VaR}_{t+1} = \mu_{p,t+1} + \sigma_{p,t+1} q, \tag{21}$$

where q is the ϑ-quantile of the distribution of $z_{p,t+1}$. Clearly, the closed form expressions for the vector of bond portfolio returns and their covariance matrix in (12) and (13), respectively, can be readily applied to the computation of the bond portfolio VaR in (21).

3 Estimation procedure

In this section, we present the estimation procedure for the parameters of the yield curve and of volatility models. The estimation is performed in a multi-step procedure in which the parameters of the factor model are first estimated, and resulting residuals are used to estimate the volatility models discussed in Section 2.2.

3.1 Estimation of the yield curve models

The alternative yield curve specifications considered in the paper are all nested and can therefore be captured in one general formulation given by the system of equations (1) and (2). The most straightforward approach to estimate the factors and parameters of this system consists of a two-step procedure proposed by Diebold & Li [2006], where the parameter λ_t is treated as fixed. This treatment greatly simplifies the estimation procedure, after fixing λ_t, it is trivial to estimate β_{1t}, β_{2t}, and β_{3t} from equation (4) via ordinary least squares (OLS) regressions. In the first step, the measurement equation is treated as a cross section for each period of time, and ordinary least squares (OLS) is employed to estimate the factors for all time periods individually. Given the estimated time-series for the factors, the second step then consists of modeling the dynamics of the factors in (2) by fitting either a joint VAR(1) model, or by estimating separate AR(1) models. To simplify the estimation procedure, Diebold & Li [2006] suggest reducing the parameter vector by setting the value of λ_t on a priori specified value, which is held fixed, rather than treating it as an unknown parameter.

The decay parameters are estimated by minimizing the sum of squared fitting errors of the model. That is, for a given set of estimated parameters the model-implied yields $y_t(\tau) = \Lambda(\lambda) f_t$ are computed, and then the sum

$$Z = \sum_{t=1}^{T} \sum_{i=1}^{N} \left(\hat{y}_t(\tau_i) - y_t(\tau_i)\right)^2$$

is minimized with respect to λ_1 and λ_2. More specifically, λ is chosen to minimize the difference between the adjusted yield, $\widehat{y}_t$, and the observed yield, y_t. Although being possibly less efficient than a joint estimation of all model parameters in a one-step maximum likelihood procedure, the two-step approach has the clear advantage that it is fast and thus much better suited for the recursive out-of-sample forecast exercise carried out in this paper.

3.2 Estimation of the covariance matrix of bond yields

To obtain the conditional covariance matrix of the factors, $\Omega_{t|t-1}$, a DCC specification in (6) is used. The estimation of the DCC model can be conveniently divided into volatility part and correlation part. The volatility part refers to estimating the univariate conditional volatility models of the factors using a GARCH-type specification. The parameters of univariate volatility models are estimated by quasi maximum likelihood (QML) assuming Gaussian innovations.[3] The correlation part refers to the estimation of the conditional correlation matrix in (7) and (8). To estimate the parameters of the correlation matrix, we employ the composite likelihood (CL) method proposed by Engle *et al.* [2008]. As pointed out by Engle *et al.* [2008], the CL estimator provides more accurate parameter estimates in comparison to the two-step procedure proposed by Engle & Sheppard [2001] and Sheppard [2003], especially in large problems.

4 Empirical application

In order to illustrate the applicability of the proposed estimators for the vector of expected bond returns and their conditional covariance matrix, in this Section we consider the problem of the one-step-ahead VaR forecasting of an equally-weighted bond portfolio as discussed in Sections 2.3

[3] A review of issues related to the estimation of univariate GARCH models, such as the choice of initial values, numerical algorithms, accuracy, and asymptotic properties are given by Berkes *et al.* [2003], Robinson & Zaffaroni [2006], Francq & Zakoian [2009] and Zivot [2009]. It is important to note that even when the normality assumption is inappropriate, the QML estimator of univariate GARCH models based on maximizing the Gaussian likelihood is consistent and asymptotically normal, provided that the conditional mean and variance of the GARCH model are correctly specified, see Bollerslev & Wooldridge [1992].

and 2.4. As we noted above, our interest is on the portfolio VaR for a long position in which traders have bought fixed income securities and wish to measure the risk associated to a decrease in their market prices.

4.1 Methodology for backtesting the VaR

A important issue related to VaR modeling is the backtesting, which is the analysis of past VaR violations, see Christoffersen [1998], Christoffersen *et al.* [2001], and Andersen *et al.* [2006]. This analysis is based on the hit sequence, which is a sequence of binary variables that denotes VaR violations and can be defined as

$$I_t = \begin{cases} 1 & \text{if} \ \ r_{p,t} < VaR_t \\ 0 & \text{if} \ \ r_{p,t} > VaR_t \end{cases},$$

where $r_{p,t}$ is the bond portfolio return at time t. Clearly, the behavior of the hit sequence is of main interest. Risk managers are concerned with VaR violations and, equally importantly, with whether these violations are clustered in time or if they appear to be randomly sparse. Clustered violations indicate that the VaR model can be misspecified and can fail to predict the portfolio VaR in times of high volatility such as during financial crises. For risk measurement purposes, the accuracy of VaR estimates during financial turmoils is highly desirable. Christoffersen [1998] point out that the problem of determining the accuracy of the VaR can be reduced to the problem of determining whether the hit sequence satisfies two properties. The first is the unconditional coverage property, which states that the probability of realizing a loss in excess of the reported VaR must be precisely $\vartheta \times 100\%$. To check if this is the case, one has to compute the empirical (or realized) hit rate, which is the number of times in which the observed bond portfolio returns is lower than the estimated VaR over the total number of periods analyzed, i.e. $\text{hit rate} = \frac{1}{T}\sum_{t=1}^{T} I\left(r_{p,t} < VaR_t\right)$. For instance, when computing the VaR at the 1% nominal level, one would expect that in 1% of the cases the observed portfolio return should be lower than the estimated VaR (i.e. a hit rate of 1%). The second aspect is the independence property, which indicates whether two elements of the hit sequence are independent from each other. Intuitively, if previous VaR violations presage a future VaR violation then this points to a general inadequacy in the reported VaR measure.

In this paper we employ the independence, unconditional and conditional coverage tests proposed by Christoffersen [1998]. To test the independence in the hit sequence, a likelihood ratio test (LR) of the following form is conducted,

$$LR_{ind} = -2\log\left[L\left(\hat{\Pi}_2; I_1, I_2, ..., I_T\right)/L\left(\hat{\Pi}_1; I_1, I_2, ..., I_T\right)\right], \tag{22}$$

where the numerator corresponds to the likelihood function of a first order Markov model estimated with the output sequence $\{\hat{I}_t\}$ obtained from the model and the denominator is the likelihood function of a binary Markov chain, and $\hat{\Pi}_1$ and $\hat{\Pi}_2$ are the corresponding transition probability matrices given by

$$\hat{\Pi}_1 = \left[\begin{array}{cc} \frac{n_{00}}{n_{00}+n_{01}} & \frac{n_{01}}{n_{00}+n_{01}} \\ \frac{n_{10}}{n_{10}+n_{11}} & \frac{n_{11}}{n_{10}+n_{11}} \end{array} \right], \tag{23}$$

and $\hat{\Pi}_2 = (n_{01} + n_{11}) / (n_{00} + n_{10} + n_{01} + n_{11})$, where n_{ij} is the number of observations with value i followed by j. The LR_{ind} test is asymptotically distributed as a χ^2 distribution with 1 degree of freedom. To test the unconditional coverage, the null hypothesis is $E[I] = \vartheta$ against the alternative $E[I] \neq \vartheta$. The likelihood function under the null is $L(p; I_1, I_2, ..., I_T) = (1-\vartheta)^{n_0}(\vartheta)^{n_1}$ and under the alternative is $L(\pi; I_1, I_2, ..., I_T) = (1-\pi)^{n_0}(\pi)^{n_1}$. The resulting LR test is $LR_{uc} = -2\log[L(p; I_1, I_2, ..., I_T)/L(\hat{\pi}; I_1, I_2, ..., I_T)]$, where $\hat{\pi} = n_1/(n_0 + n_1)$ and n_1 and n_0 are the number of occurrences of ones and zeros in the hit sequence, respectively. The LR_{uc} is distributed as a χ^2 distribution with 1 degree of freedom. Finally, to test the joint hypothesis of independence and unconditional coverage, a LR of the form is conducted,

$$LR_{cc} = 2\log[L(p; I_1, I_2, ..., I_T)/L(\hat{\Pi}_1; I_1, I_2, ..., I_T)],$$

which is refereed to as the conditional coverage test. The LR_{cc} test is distributed as a a χ^2 distribution with 2 degrees of freedom.

It is worth noting that the independence, unconditional coverage, and conditional coverage tests, though appropriate to evaluate the accuracy of a single model, may not appropriate for ranking alternative estimates of the VaR and can provide an ambiguous decision about which candidate model is better. Therefore, it is interesting to enhance the backtesting analysis by using statistical tests designed to evaluate the comparative performance among candidate models, see Santos *et al.* [2012a] for a discussion. In this sense, we follow Santos *et al.* [2012a] and consider the comparative predictive ability (CPA) test proposed by Giacomini & White [2006]. Consequently, on top of evaluating whether each of the estimated VaRs are adequate, we also compare and rank them by implementing the CPA test of Giacomini & White [2006] which can be applied to the comparison between nested and non-nested models and among several alternative estimation procedures. The CPA test is implemented using the following asymmetric linear (tick) loss function of order ϑ:

$$\mathcal{L}^{\vartheta}(e_t) = (\vartheta - I(e_t < 0))\, e_t, \tag{24}$$

where $e_t = \text{VaR}_t^{\vartheta} - r_{p,t}$. The loss function in (24) is the implicit loss function whenever the object of interest is a forecast of a particular ϑ-quantile; see Giacomini & Komunjer [2005]. Consequently, finding the model that minimizes (24) is an intuitive and appealing criterion to compare predictive ability. A Wald-type test is conducted as follows:

$$CPA^{\vartheta} = T\left(T^{-1}\sum_{t=1}^{T-1}\mathcal{I}_t\ \mathcal{LD}_{t+1}^{\vartheta}\right)'\hat{\Theta}^{-1}\left(T^{-1}\sum_{t=1}^{T-1}\mathcal{I}_t\ \mathcal{LD}_{t+1}^{\vartheta}\right), \tag{25}$$

where T is the sample size, $\mathcal{LD}_t^{\vartheta}$ is the loss difference between the two models, and $\hat{\Theta}$ is a matrix that consistently estimate the variance of $\mathcal{I}_t\ \mathcal{LD}^{\vartheta}_{t+1}$. Following Giacomini & White [2006], we assume $\mathcal{I}_t = (1,\ \mathcal{LD}^{\vartheta}_t)$. The null hypothesis of equal predictive ability is rejected for a size ξ when $CPA^{\vartheta} > \chi^2_{T,1-\xi}$.

4.2 Data

Our data set consists of time series of yields of Brazilian Inter Bank Deposit Future Contract (DI-futuro), which is one of the largest fixed-income markets among emerging economies, collected on a daily basis. The DI-futuro contract with maturity τ is a zero-coupon future contract in which the underlying asset is the DI-futuro interest rate accrued on a daily basis, capitalized between trading period t and τ.[4] The contract value is set by its value at maturity, $R\$100,000.00$, discounted according to the accrued interest rate negotiated between the seller and the buyer. A similar data set is also used by Almeida & Vicente [2009].

The Brazilian Mercantile and Futures Exchange (BM&F) is the entity that offers the DI-futuro contract and determines the number of maturities with authorized contracts. In general, there are around 20 maturities with authorized contracts every day. In 2010 the DI-futuro market traded a total of 293 million contracts corresponding to US$ 15 billion. The DI-futuro contract is very similar to the zero-coupon bond, except for the daily payment of marginal adjustments. Every day the cash flow is the difference between the adjustment price of the current day and the adjustment price of the previous day, indexed by the DI-futuro rate of the previous day.

We use time series of daily closing yields of the DI-futuro contracts with highest liquidity ranging from January 2006 to December 2010 ($T = 986$ observations). In practice, contracts with all maturities are not observed on a daily basis. Therefore, based on the observed rates for the available maturities, the data were converted into fixed maturities of 1, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 42 and 48 months, using the cubic splines interpolation method originally proposed by McCulloch [1971, 1975][5].

Table 1 reports descriptive statistics for the Brazilian interest rate yield curve based on the DI-futuro market. The table reports the mean, standard deviation, minimum, maximum and the sample autocorrelations at lags of one day, one week, and one month. The summary statistics confirm the presence of stylized facts common to yield curve data: the average curve is upward sloping and concave, volatility is decreasing with maturity, autocorrelations are very high specially

[4] The DI-futuro rate is the average daily rate of Brazilian interbank deposits (borowing/lending), calculated by the Clearinghouse for Custody and Settlements (CETIP) for all business days. The DI-futuro rate, which is published on a daily basis, is expressed in annually compounded terms, based on 252 business days. When buying a DI-futuro contract for the price at time t and keeping it until maturity τ, the gain or loss is given by:

$$100.000\left(\frac{\prod_{i=1}^{\zeta(t,\tau)}(1+y_i)^{\frac{1}{252}}}{(1+DI^*)^{\frac{\zeta(t,\tau)}{252}}}-1\right),$$

where y_i denotes the DI-futuro rate, $(i-1)$ days after the trading day. The function $\zeta(t,\tau)$ represents the number of working days between t and τ.

[5] For further details and applications of this method, see Hagan & West [2006] and Hayden & Ferstl [2010].

for shorter maturities.

Table 1: **Descriptive Statistics**

The table reports summary statistics for the Brazilian yield curve based on DI-futuro contracts. The sample consists of daily yield data from January 2006 to December 2010. Maturities are measured in months. We show for each maturity the mean, standard deviation, minimum, maximum, skewness and excess of kurtosis. The last three columns contains the autocorrelations with a lag of one day, one week and one month, respectively (ACF, $\widehat{\rho}(1)$, $\widehat{\rho}(5)$, $\widehat{\rho}(21)$).

Maturity	τ	Mean	Standard Deviation	Minimum	Maximum	Skewness	Kurtosis	$\widehat{\rho}(1)$	$\widehat{\rho}(5)$	$\widehat{\rho}(21)$
Month	3	10.82	1.65	8.58	14.34	0.220	2.006	0.999	0.997	0.969
	6	10.88	1.67	8.59	14.52	0.264	2.071	0.999	0.997	0.968
	9	10.94	1.69	8.58	14.69	0.306	2.132	0.999	0.996	0.967
	12	11.09	1.72	8.61	15.32	0.386	2.241	0.999	0.995	0.961
	15	11.34	1.73	8.73	16.04	0.495	2.373	0.998	0.992	0.950
	18	11.60	1.72	8.99	16.40	0.572	2.461	0.998	0.989	0.938
	21	11.85	1.68	9.35	16.92	0.655	2.565	0.997	0.986	0.925
	24	12.04	1.61	9.55	17.12	0.718	2.659	0.996	0.982	0.911
	27	12.21	1.55	9.79	17.26	0.805	2.815	0.995	0.979	0.894
	30	12.33	1.49	10.06	17.44	0.912	3.026	0.995	0.975	0.877
	33	12.43	1.45	10.27	17.62	1.005	3.290	0.994	0.972	0.859
	36	12.50	1.41	10.42	17.78	1.085	3.586	0.993	0.968	0.843
	42	12.60	1.32	10.71	17.83	1.281	4.180	0.992	0.961	0.814
	48	12.68	1.24	11.09	17.93	1.465	4.910	0.990	0.955	0.788

Figure 1 displays a three-dimensional plot of the data set and illustrates how yield levels and spreads vary substantially throughout the sample. The plot also suggests the presence of an underlying factor structure. Although the yield series vary heavily over time for each of the maturities, a strong common pattern in the 15 series over time is apparent. For most months, the yield curve is an upward sloping function of time to maturity. For example, last year of the sample is characterized by rising interest rates, especially for the shorter maturities, which respond faster to the contractionary monetary policy implemented by the Brazilian Central Bank in the first half of 2010. It is clear from Figure 1 that not only the level of the term structure fluctuates over time but also its slope and curvature. The curve takes on various forms ranging from nearly flat to (inverted) S-type shapes.

4.3 Benchmark models

In order to provide evidence of the flexibility of the proposed approach discussed in Section 2, we implement a set of alternative specifications to model the conditional covariance matrix of bond returns. We consider alternative conditional correlation specification to model the covariance matrix of the factors in (6). Our first benchmark specification is the constant conditional correlation (CCC) model of Bollerslev [1990]. In this case, we consider that the correlation matrix Ψ_t in (6) is constant over time. Our second benchmark specification is the dynamic equicorrelation (DECO) model proposed by Engle & Kelly [2009], which belongs to the class of conditional correlation model but uses a more parsimonious specification in comparison to the DCC model discussed above. In the DECO model, the conditional correlation matrix Ψ_t is given by:

$$\Psi_t = \Psi_t^{DECO},$$

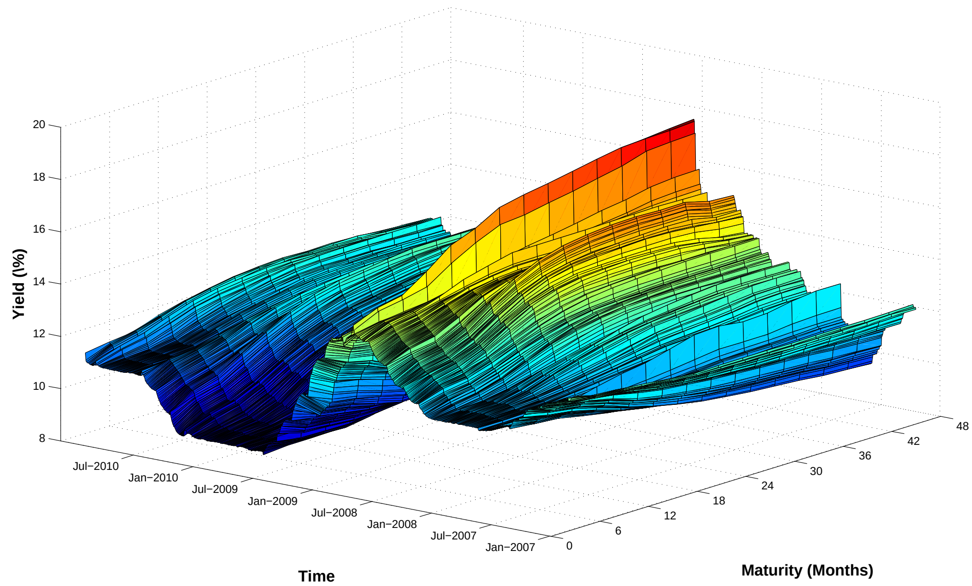

Figure 1: **Evolution of the yield curve**
The figure plots the evolution of term structure of interest rates (based on DI-futuro contracts) for the time horizon of 2006:01-2010:12. The sample consisted of the daily yields for the maturities of 1, 3, 4, 6, 9, 12, 15, 18, 24, 27, 30, 36, 42 and 48 months.

where Ψ_t^{DECO} is the conditional equicorrelation matrix defined as:

$$\Psi_t^{DECO} = (1 - \psi_t^{equi})I_n + \psi_t^{equi} J_n,$$

where ψ_t^{equi} is the equicorrelation at time t, I_n is a N-dimensional identity matrix, and J_n is a $N \times N$ matrix of ones. Following Engle & Kelly [2009], the DECO sets the equicorrelation ψ_t^{equi} equal to the average pairwise DCC correlation. The model is estimated using the two-step procedure proposed by Engle & Sheppard [2001].

Our second class of benchmark specifications to compute the VaR uses the same expressions for vector of expected bond returns and for the covariance matrix of bond returns discussed in Section 2 but replaces the matrices Σ_t and Ω_t in (1) and (2), respectively, with their sample counterparts. In order words, this benchmark specification considers that Σ_t and Ω_t are sample covariance matrices. This is an interesting benchmark to our approach since we originally adopt more sophisticated multivariate and univariate GARCH-type specifications to model these two matrices.

4.4 Implementation details

All models are estimated using a recursive expanding estimation window. Departing from the first 500 observations, all models are estimated and their corresponding one-step-ahead VaR estimated are obtained using (21). Next, we add one observation to the estimation window and re-estimate all models and obtain another one-step-ahead estimate of the VaR. This process is repeated until the end of the data set is reached. In the end, we obtain 486 out-of-sample one-step-ahead VaR forecasts. All results discussed in Section 4.5 are based solely on out-of-sample observations.

4.5 Results

In this Section, we report the backtesting results of the VaR estimates obtained with the specifications for the vector of expected bond returns and their conditional covariance matrix proposed in Section 2.3. The computation of the VaR is discussed in Section 2.4 whereas the methodology for backtesting is discussed in Section 4.1. In order to facilitate the exposition of the results, we denote by NS-AR-DCC and NS-VAR-DCC the the VaR estimates obtained when the 3-factor Nelson-Siegel model is used to model bond yields and an AR(1) and VAR(1) specifications, respectively, are used to model the factor dynamics and a DCC-GARCH specification is used to model the covariance matrix of the factors. Similarly, we denote by Svensson-AR-DCC and Svensson-VAR-DCC the VaR estimates obtained when the 4-factor Svensson model is used to model bond yields and an AR(1) and VAR(1) specifications, respectively, are used to model the factor dynamics and a DCC-GARCH specification is used to model the covariance matrix of the factors. We also consider the cases in which the covariance matrix of the factors in (2) is modeled according to a CCC and DECO specifications. Finally, we consider the case in which the covariance matrix of the factors in (2) and the covariance matrix of the residuals from the yield model in (1) are sample estimates. Details of these benchmark specifications are given in Section 4.3.

Table 2 reports the the hit rate and the p-values of the independence, unconditional coverage and conditional coverage tests for the 1%, 2.5%, and 5% VaR estimates obtained with each of the specifications proposed in the paper. We find that some specifications passed *all* backtests in the thee VaR levels considered. For instance, when looking at the VaR estimates for the 1% level, we observe that the NS-VAR-DCC and NS-VAR-CCC passed all backtests. These specifications achieved an exact empirical coverage rate of 1%. The VaR estimates delivered by the Svensson-AR-DCC specification also performed well, with an empirical coverage rate of 1.2%. As for the VaR estimates for the 2.5% level, we find once again that the NS-VAR-DCC and NS-VAR-CCC performed remarkably well as they passed all backtests. Finally, as for the VaR estimates for the 5% level we find that five specifications passed the correct unconditional coverage tests, but only the NS-VAR-DCC and NS-VAR-CCC specifications passed joint conditional coverage test. This results suggest that these specifications deliver very accurate VaR estimates for the bond portfolio considered in the paper and provide favorable evidence to the proposed estimators for the vector of expected bond return and their covariance matrix proposed in the paper.

We also report in Table 3 the p-values of the Giacomini & White [2006] CPA test for each pairwise comparison among all specifications considered in the paper. In order to facilitate the interpretation of the results, we highlight in bold the cases in which the model in the line outperforms the model in the column. We observe that the results in Table 3 corroborate the backtesting results discussed above. The NS-VAR-DCC specification outperform all other specifications in all cases. In few cases, however, the differences in performance with respect to other specifications are not significant. For instance, the difference with respect to the NS-VAR-CCC specification is not statistically significant. This suggests that both NS-VAR-DCC and NS-VAR-CCC performed very well in modeling the VaR for the bond portfolio.

In order to further illustrate the results presented in Tables 2 to 3, we plot in Figure 2 the returns of the equally-weighted bond portfolio over the out-of-sample period and the 1%-VaR estimates delivered by the NS-AR-DCC (upper right), NS-VAR-DCC (upper left), Svensson-AR-DCC (lower right), and Svensson-VAR-DCC (lower left) specifications. We observe that the VaR estimates obtained when the AR(1) specification is used to model the factor dynamics tend bo be more noisy than those obtained with a VAR(1) specifications. Moreover, the Figure shows we find the the dynamic Svensson yield curve model tend to generate more conservative VaR estimates in comparison to the dynamic Nelson-Siegel model. This corroborates the findings in Table 2 which reveals that the empirical coverage rates obtained with the Svensson model tend to be lower than those obtained with the Nelson-Siegel model.

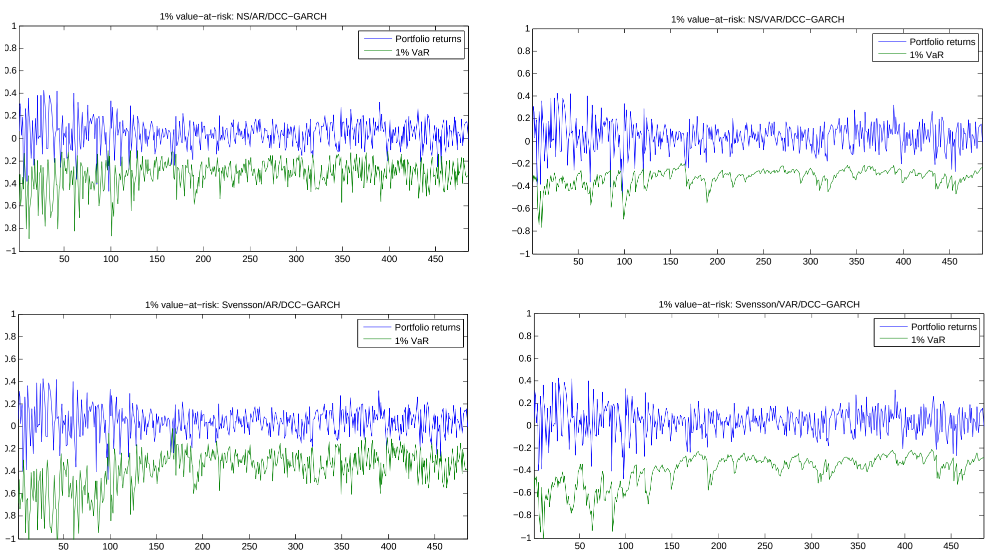

Figure 2: **Value-at-risk (VaR) estimates**

5 Concluding remarks

Obtaining accurate risk measures can be seen as an important issue in risk management. In this sense, the use of VaR as a risk measure plays a major role in monitoring market risk exposure and determining the amount of capital subject to regulatory control. The vast majority of the existing evidence on VaR modeling focus on measuring the risk of equity portfolios. Therefore, in this paper we amend the literature on VaR-based risk measurement by putting forward a novel approach to measure risk in bond portfolios. Our approach significantly differs from the existing ones as it is built upon a general class of well established term structure factor models such as the dynamic version of the Nelson-Siegel model proposed by Diebold & Li [2006], and the four factor version proposed by Svensson [1994]. We derive closed-form expressions for the vector expected bond returns and for the covariance matrix of bond returns based on yield curve models to compute the VaR of a bond portfolio.

We provide an empirical application by considering a data set composed of constant-maturity future contracts of the Brazilian Inter Bank Deposit Future Contract (DI-futuro) which is equivalent to a zero-coupon bond and is highly liquid. Based on the estimates for the vector of expected returns of these fixed-income assets and their conditional covariance matrix, we obtain out-of-sample VaR estimates for an equally-weighted bond portfolio and provide a comprehensive backtesting analysis. Our results indicate that the proposed specifications outperform several benchmark specifications in modeling and forecasting the one-step-ahead VaR at different levels.

Table 2: Backtesting results

The table reports the backtesting results for the VaR estimates at the $\vartheta = 1\%$, $\vartheta = 2.5\%$, and $\vartheta = 5\%$ levels obtained with the specifications for the vector of expected bond returns and their conditional covariance matrix proposed in the paper. We report the hit rate and the p-values of the independence ("Indep."), unconditional coverage ("U.C.") and conditional coverage ("C.C.") tests. NS-AR-DCC and NS-VAR-DCC denote the the VaR estimates obtained when the 3-factor Nelson-Siegel model is used to model bond yields and an AR(1) and VAR(1) specifications, respectively, are used to model the factor dynamics and a DCC-GARCH specification is used to model the covariance matrix of the factors. Similarly, Svensson-AR-DCC and Svensson-VAR-DCC denote the VaR estimates obtained when the 4-factor Svensson model is used to model bond yields. A similar notation applies to the remaining specifications. We highlight in bold the p-values indicating the non-rejection of the null hypothesis of the test.

Yield curve model	Factor dynamics	Covariance specification	Hit rate	Indep.	U.C.	C.C.
				$\vartheta = 1\%$		
Nelson-Siegel	AR(1)	DCC-GARCH	2.5%	**0.293**	0.006	0.013
Nelson-Siegel	AR(1)	CCC-GARCH	2.7%	0.042	0.002	0.001
Nelson-Siegel	AR(1)	DECO-GARCH	0.0%	0.000	0.000	0.000
Nelson-Siegel	AR(1)	Sample	0.6%	0.009	**0.366**	0.023
Nelson-Siegel	VAR(1)	DCC-GARCH	1.0%	**0.759**	**0.942**	**0.951**
Nelson-Siegel	VAR(1)	CCC-GARCH	1.0%	**0.759**	**0.942**	**0.951**
Nelson-Siegel	VAR(1)	DECO-GARCH	0.0%	0.000	0.000	0.000
Nelson-Siegel	VAR(1)	Sample	0.0%	0.000	0.000	0.000
Svensson	AR(1)	DCC-GARCH	1.2%	0.056	**0.609**	**0.141**
Svensson	AR(1)	CCC-GARCH	0.8%	0.020	**0.692**	0.062
Svensson	AR(1)	DECO-GARCH	0.0%	0.000	0.000	0.000
Svensson	AR(1)	Sample	0.6%	0.009	**0.366**	0.023
Svensson	VAR(1)	DCC-GARCH	0.0%	0.000	0.000	0.000
Svensson	VAR(1)	CCC-GARCH	0.0%	0.000	0.000	0.000
Svensson	VAR(1)	DECO-GARCH	0.0%	0.000	0.000	0.000
Svensson	VAR(1)	Sample	0.0%	0.000	0.000	0.000
				$\vartheta = 2.5\%$		
Nelson-Siegel	AR(1)	DCC-GARCH	4.3%	0.001	0.019	0.000
Nelson-Siegel	AR(1)	CCC-GARCH	3.3%	0.012	**0.279**	0.024
Nelson-Siegel	AR(1)	DECO-GARCH	0.2%	**0.963**	0.000	0.000
Nelson-Siegel	AR(1)	Sample	1.2%	0.000	0.049	0.000
Nelson-Siegel	VAR(1)	DCC-GARCH	1.6%	**0.615**	**0.204**	**0.393**
Nelson-Siegel	VAR(1)	CCC-GARCH	1.4%	**0.662**	**0.107**	**0.249**
Nelson-Siegel	VAR(1)	DECO-GARCH	0.0%	0.000	0.000	0.000
Nelson-Siegel	VAR(1)	Sample	0.8%	**0.809**	0.006	0.023
Svensson	AR(1)	DCC-GARCH	2.3%	0.020	**0.745**	0.063
Svensson	AR(1)	CCC-GARCH	1.9%	0.008	**0.345**	0.019
Svensson	AR(1)	DECO-GARCH	0.0%	0.000	0.000	0.000
Svensson	AR(1)	Sample	0.8%	0.020	0.006	0.002
Svensson	VAR(1)	DCC-GARCH	0.2%	**0.963**	0.000	0.000
Svensson	VAR(1)	CCC-GARCH	0.0%	0.000	0.000	0.000
Svensson	VAR(1)	DECO-GARCH	0.0%	0.000	0.000	0.000
Svensson	VAR(1)	Sample	0.4%	**0.911**	0.000	0.001
				$\vartheta = 5\%$		
Nelson-Siegel	AR(1)	DCC-GARCH	6.6%	0.000	**0.120**	0.000
Nelson-Siegel	AR(1)	CCC-GARCH	6.6%	0.000	**0.120**	0.000
Nelson-Siegel	AR(1)	DECO-GARCH	0.6%	**0.859**	0.000	0.000
Nelson-Siegel	AR(1)	Sample	2.3%	0.001	0.002	0.000
Nelson-Siegel	VAR(1)	DCC-GARCH	3.5%	**0.273**	**0.113**	**0.157**
Nelson-Siegel	VAR(1)	CCC-GARCH	3.3%	**0.303**	0.069	**0.113**
Nelson-Siegel	VAR(1)	DECO-GARCH	0.2%	**0.963**	0.000	0.000
Nelson-Siegel	VAR(1)	Sample	1.2%	0.056	0.000	0.000
Svensson	AR(1)	DCC-GARCH	4.9%	0.000	**0.967**	0.000
Svensson	AR(1)	CCC-GARCH	3.9%	0.000	**0.261**	0.001
Svensson	AR(1)	DECO-GARCH	0.2%	**0.963**	0.000	0.000
Svensson	AR(1)	Sample	1.9%	0.000	0.000	0.000
Svensson	VAR(1)	DCC-GARCH	2.1%	0.526	0.001	0.003
Svensson	VAR(1)	CCC-GARCH	1.6%	0.615	0.000	0.000
Svensson	VAR(1)	DECO-GARCH	0.0%	0.000	0.000	0.000
Svensson	VAR(1)	Sample	1.0%	0.036	0.000	0.000

Table 3: p-values of the conditional predictive ability (CPA) test

The table reports the p-values of the conditional predictive ability (CPA) test of Giacomini and White (2006) for each pairwise comparison among VaR models and for the three VaR levels considered ($\vartheta = 1\%$, $\vartheta = 2.5\%$, and $\vartheta = 5\%$). We highlight in bold the instances in which the model in the line outperforms the model in the column.

$\vartheta = 1\%$	NS-AR-CCC	NS-AR-DECO	NS-AR-Sample	NS-VAR-DCC	NS-VAR-CCC	NS-VAR-DECO	NS-VAR-Sample	SV-AR-DCC	SV-AR-CCC	SV-AR-DECO	SV-AR-Sample	SV-VAR-DCC	SV-VAR-CCC	SV-VAR-DECO	SV-VAR-Sample
NS-AR-DCC	**0.536**	**0.079**	**0.027**	0.357	0.322	**0.134**	**0.512**	0.632	0.792	**0.000**	**0.006**	0.388	0.681	**0.001**	**0.322**
NS-AR-CCC		**0.056**	**0.004**	0.282	0.232	**0.058**	0.282	0.329	0.813	**0.000**	**0.000**	0.279	0.391	**0.000**	**0.186**
NS-AR-DECO			**0.663**	0.000	0.000	**0.565**	0.000	0.000	**0.001**	**0.000**	**0.855**	0.000	0.000	**0.000**	0.000
NS-AR-Sample				0.005	0.002	0.727	0.244	0.024	0.140	**0.365**	0.569	0.015	0.046	**0.447**	0.385
NS-VAR-DCC					**0.545**	**0.000**	**0.002**	**0.380**	**0.118**	**0.000**	**0.000**	**0.232**	**0.093**	**0.000**	**0.001**
NS-VAR-CCC						**0.000**	**0.000**	**0.252**	**0.049**	**0.000**	**0.000**	**0.232**	**0.062**	**0.000**	**0.000**
NS-VAR-DECO							0.000	0.000	0.002	**0.000**	**0.720**	0.000	0.000	**0.000**	0.000
NS-VAR-Sample								0.105	0.359	**0.000**	**0.208**	0.000	0.000	**0.000**	**0.000**
SV-AR-DCC									**0.007**	**0.000**	**0.020**	0.224	0.702	**0.000**	**0.019**
SV-AR-CCC										**0.000**	**0.069**	0.051	0.319	**0.000**	**0.118**
SV-AR-DECO											0.221	0.000	0.000	0.324	0.000
SV-AR-Sample												0.004	0.022	**0.267**	0.388
SV-VAR-DCC													0.000	**0.000**	**0.000**
SV-VAR-CCC														**0.000**	**0.000**
SV-VAR-DECO															0.000
$\vartheta = 2.5\%$	NS-AR-CCC	NS-AR-DECO	NS-AR-Sample	NS-VAR-DCC	NS-VAR-CCC	NS-VAR-DECO	NS-VAR-Sample	SV-AR-DCC	SV-AR-CCC	SV-AR-DECO	SV-AR-Sample	SV-VAR-DCC	SV-VAR-CCC	SV-VAR-DECO	SV-VAR-Sample
NS-AR-DCC	**0.622**	**0.013**	**0.009**	0.095	0.118	**0.024**	**0.507**	0.654	**0.994**	**0.000**	**0.003**	0.155	0.395	**0.000**	**0.245**
NS-AR-CCC		**0.020**	**0.001**	0.087	0.103	**0.026**	0.416	0.323	0.640	**0.000**	**0.000**	0.128	0.302	**0.000**	**0.233**
NS-AR-DECO			**0.511**	0.000	0.000	0.568	0.000	0.000	0.001	**0.000**	**0.811**	0.000	0.000	**0.000**	0.000
NS-AR-Sample				0.000	0.000	0.578	0.137	0.000	0.003	0.017	0.091	0.000	0.003	**0.029**	0.283
NS-VAR-DCC					**0.300**	**0.000**	**0.000**	**0.109**	**0.017**	**0.000**	**0.000**	**0.121**	**0.046**	**0.000**	**0.000**
NS-VAR-CCC						**0.000**	**0.000**	**0.124**	**0.017**	**0.000**	**0.000**	**0.126**	**0.051**	**0.000**	**0.000**
NS-VAR-DECO							0.000	0.002	0.008	**0.000**	**0.876**	0.000	0.000	**0.000**	0.000
NS-VAR-Sample								0.311	0.457	**0.000**	**0.081**	0.000	0.000	**0.000**	**0.000**
SV-AR-DCC									**0.001**	**0.000**	**0.000**	0.082	0.410	**0.000**	**0.125**
SV-AR-CCC										**0.000**	**0.002**	0.008	0.103	**0.000**	**0.268**
SV-AR-DECO											0.054	0.000	0.000	0.419	0.000
SV-AR-Sample												0.000	0.001	**0.081**	0.242
SV-VAR-DCC													**0.000**	**0.000**	**0.000**
SV-VAR-CCC														**0.000**	**0.000**
SV-VAR-DECO															0.000
$\vartheta = 5\%$	NS-AR-CCC	NS-AR-DECO	NS-AR-Sample	NS-VAR-DCC	NS-VAR-CCC	NS-VAR-DECO	NS-VAR-Sample	SV-AR-DCC	SV-AR-CCC	SV-AR-DECO	SV-AR-Sample	SV-VAR-DCC	SV-VAR-CCC	SV-VAR-DECO	SV-VAR-Sample
NS-AR-DCC	**0.350**	**0.000**	**0.000**	0.021	0.030	**0.001**	**0.259**	0.426	**0.393**	**0.000**	**0.000**	0.112	0.236	**0.000**	**0.141**
NS-AR-CCC		**0.000**	**0.000**	0.020	0.027	**0.001**	**0.316**	0.159	**0.192**	**0.000**	**0.000**	0.090	0.212	**0.000**	**0.180**
NS-AR-DECO			**0.251**	0.000	0.000	0.330	0.000	0.000	0.000	**0.000**	**0.464**	0.000	0.000	**0.000**	0.000
NS-AR-Sample				0.000	0.000	0.457	0.120	0.000	0.001	**0.001**	**0.022**	0.000	0.000	**0.001**	0.276
NS-VAR-DCC					**0.476**	**0.000**	**0.000**	**0.029**	**0.004**	**0.000**	**0.000**	**0.027**	**0.008**	**0.000**	**0.000**
NS-VAR-CCC						**0.000**	**0.000**	**0.035**	**0.004**	**0.000**	**0.000**	**0.029**	**0.008**	**0.000**	**0.000**
NS-VAR-DECO							0.000	0.000	0.003	**0.000**	**0.634**	0.000	0.000	**0.000**	0.000
NS-VAR-Sample								0.433	0.607	**0.000**	**0.037**	0.000	0.000	**0.000**	**0.003**
SV-AR-DCC									**0.000**	**0.000**	**0.000**	0.052	0.200	**0.000**	**0.189**
SV-AR-CCC										**0.000**	**0.000**	0.005	0.034	**0.000**	**0.345**
SV-AR-DECO											0.002	0.000	0.000	0.580	0.000
SV-AR-Sample												0.000	0.000	**0.006**	0.152
SV-VAR-DCC													**0.000**	**0.000**	**0.000**
SV-VAR-CCC														**0.000**	**0.000**
SV-VAR-DECO															0.000

References

Aguilar, O., & West, M. 2000. Bayesian dynamic factor models and portfolio allocation. *Journal of Business and Economic Statistics*, **18**(3), 338–357.

Almeida, C., & Vicente, J. 2009. Are interest rate options important for the assessment of interest rate risk? *Journal of Banking & Finance*, **33**(8), 1376–1387.

Andersen, T.G., Bollerslev, T., Christoffersen, P.F., & Diebold, F.X. 2006. *Volatility and correlation forecasting*. Vol. 1.

Andersen, Torben G., & Benzoni, Luca. 2010 (Feb.). *Stochastic Volatility*. CREATES Research Papers 2010-10. School of Economics and Management, University of Aarhus.

Bauwens, L., Laurent, S., & Rombouts, J.V.K. 2006. Multivariate GARCH models: a survey. *Journal of Applied Econometrics*, **21**(1), 79–109.

Berkes, I., Horváth, L., & Kokoszka, P. 2003. GARCH processes: structure and estimation. *Bernoulli*, **9**(2), 201–227.

Berkowitz, J., & O'Brien, J. 2002. How accurate are value-at-risk models at commercial banks? *The Journal of Finance*, **57**(3), 1093–1111.

Bianchi, F., Mumtaz, H., & Surico, P. 2009. The great moderation of the term structure of UK interest rates. *Journal of Monetary Economics*, **56**(6), 856–871.

BIS. 2005. *Zero-coupon yield curves: technical documentation*. Tech. rept. Bank for International Settlements.

Bollerslev, T. 1986. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, **31**(3), 307–327.

Bollerslev, T. 1990. Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH model. *Review of Economics and Statistics*, **72**(3), 498–505.

Bollerslev, T., & Wooldridge, J.M. 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. *Econometric reviews*, **11**(2), 143–172.

Brooks, C., & Persand, G. 2003. Volatility forecasting for risk management. *Journal of Forecasting*, **22**(1), 1–22.

Caldeira, J. F., Laurini, M. P., & Portugal, M. S. 2010a. Bayesian inference applied to dynamic Nelson-Siegel model with stochastic volatility. *Brazilian Review of Econometrics*, **30**(1), 123–161.

Caldeira, J. F., Moura, Guilherme V., & Portugal, Marcelo S. 2010b. Efficient yield curve estimation and forecasting in Brazil. *Economia*, **11**(1), 27–51.

Cappiello, L., Engle, R.F., & Sheppard, K. 2006. Asymmetric dynamics in the correlations of global equity and bond returns. *Journal of Financial Econometrics*, **4**(4), 537–572.

Chib, S., Omori, Y., & Asai, M. 2009. *Multivariate stochastic volatility*. Springer Verlag.

Christoffersen, P. 2009. Value-at-risk models. *In:* Andersen, T.G., Davis, R.A., Kreiss, J.-P., & Mikosch, T. (eds), *Handbook of Financial Time Series*. Springer Verlag.

Christoffersen, P., Hahn, J., & Inoue, A. 2001. Testing and comparing value-at-risk measures. *Journal of Empirical Finance*, **8**(3), 325–342.

Christoffersen, P.F. 1998. Evaluating interval forecasts. *International Economic Review*, **39**(4), 841–862.

Cox, J. C., Ingersoll, J. E., & Ross, S. A. 1985. A theory of the term structure of interest rates. *Econometrica*, **53**(2), 385–407.

de Goeij, Peter, & Marquering, Wessel. 2006. Macroeconomic announcements and asymmetric volatility in bond returns. *Journal of Banking & Finance*, **30**(10), 2659–2680.

De Pooter, M. 2007. *Examining the Nelson-Siegel class of term structure models*. Tinbergen Institute Discussion Papers. Tinbergen Institute.

DeMiguel, V., Garlappi, L., & Uppal, R. 2009. Optimal versus naive diversification: how inefficient is the 1/N portfolio strategy? *Review of Financial Studies*, **22**(5), 1915–1953.

Diebold, F., & Li, C. 2006. Forecasting the term structure of government bond yields. *Journal of Econometrics*, **130**(2), 337–364.

Diebold, F. X., & Rudebusch, G. D. 2011. *The Dynamic Nelson-Siegel Approach to Yield Curve Modeling and Forecasting*. mimeo.

Diebold, F. X., Rudebusch, G. D., & Aruoba, S. B. 2006. The macroeconomy and the yield curve: a dynamic latent factor approach. *Journal of Econometrics*, **131**(1-2), 309–338.

Ding, Z., Granger, C.W.J., & Engle, R.F. 1993. A long memory property of stock returns and a new model. *Journal of Empirical Finance*, **1**(1), 83–106.

Engle, R. 2002. Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics*, **20**(3), 339–350.

Engle, R., & Kelly, B. 2009. Dynamic equicorrelation. *NYU Working Paper No. FIN-08-038*.

Engle, R.F. 1990. Stock volatility and the crash of '87: discussion. *The Review of Financial Studies*, **3**(1), 103–106.

Engle, R.F., & Manganelli, S. 2004. CAViaR: conditional autoregressive value at risk by regression quantiles. *Journal of Business & Economic Statistics*, **22**(4), 367–382.

Engle, R.F., & Ng, V.K. 1993. Measuring and testing the impact of news on volatility. *Journal of Finance*, **48**(5), 1749–78.

Engle, R.F., & Sheppard, K. 2001. Theoretical and empirical properties of dynamic conditional correlation multivariate GARCH. *NBER Working Paper W8554*.

Engle, R.F., Shephard, N., & Sheppard, K. 2008. Fitting vast dimensional time-varying covariance models. *Discussion Paper Series n.403, Department of Economics, University of Oxford*.

Ferreira, M.A. 2005. Forecasting the comovements of spot interest rates. *Journal of International Money and Finance*, **24**(5), 766–792.

Ferreira, M.A., & Lopez, J.A. 2005. Evaluating interest rate covariance models within a value-at-risk framework. *Journal of Financial Econometrics*, **3**(1), 126–168.

Filipovic, D. 2009. *Term Structure Models.* Springer Finance.

Francq, C., & Zakoian, J.M. 2009. A tour in the asymptotic theory of GARCH estimation. *In:* Andersen, T.G., Davis, R.A., Kreiss, J.-P., & Mikosch, T. (eds), *Handbook of Financial Time Series.* Springer Verlag.

Galeano, Pedro, & Ausin, M. 2010. The Gaussian Mixture Dynamic Conditional Correlation Model: Parameter Estimation, Value at Risk Calculation, and Portfolio Selection. *Journal of Business & Economic Statistics*, **28**(4), 559–571.

Giacomini, R., & Komunjer, I. 2005. Evaluation and combination of conditional quantile forecasts. *Journal of Business & Economic Statistics*, **23**(4), 416–431.

Giacomini, R., & White, H. 2006. Tests of conditional predictive ability. *Econometrica*, **74**(6), 1545–1578.

Gimeno, Ricardo, & Nave, Juan M. 2009. A genetic algorithm estimation of the term structure of interest rates. *Computational Statistics & Data Analysis*, **53**(6), 2236–2250.

Giot, P., & Laurent, S. 2003. Value-at-risk for long and short trading positions. *Journal of Applied Econometrics*, 641–664.

Giot, P., & Laurent, S. 2004. Modelling daily value-at-risk using realized volatility and ARCH type models. *Journal of Empirical Finance*, **11**(3), 379–398.

Glosten, Lawrence, Jagannathan, Ravi, & Runkle, David. 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, **48**, 1779–1801.

Hagan, P., & West, G. 2006. Interpolation methods for curve construction. *Applied Mathematical Finance*, **13**(2), 89–129.

Harvey, A., Ruiz, E., & Shephard, N. 1994. Multivariate stochastic variance models. *Review of Economic Studies*, **61**(2), 247–64.

Haustsch, Nikolaus, & Ou, Yangguoyi. 2012. Bayesian inference in a stochastic volatility Nelson-Siegel model. *Computational Statistics and Data Analysis*, **56**(11), 3774–3792.

Hayden, Josef, & Ferstl, Robert. 2010. Zero-coupon yield curve estimation with the package termstrc. *Journal of Statistical Software*, **36**(i01), 1–34.

Jones, Charles M., Lamont, Owen, & Lumsdaine, Robin L. 1998. Macroeconomic news and bond market volatility. *Journal of Financial Economics*, **47**(3), 315–337.

Jorion, Phillip. 2006. *Value at Risk: The new benchmark for managing financial risk.* McGraw-Hill, NewYork.

Jungbacker, B., & Koopman, S.J. 2008. Likelihood-based analysis for dynamic factor models. *Tinbergen Institute Discussion Paper*. found on http://www.tinbergen.nl.

Koopman, Siem Jan, Mallee, Max I.P., & van der Wel, Michel. 2010. Analyzing the term structure of interest rates using the dynamic Nelson-Siegel model with time-varying parameters. *Journal of Business and Economic Statistics*, **28**(3), 329–343.

Korn, O., & Koziol, C. 2006. Bond portfolio optimization. *The Journal of Fixed Income*, **15**(4), 48–60.

Litterman, R., & Scheinkman, J. 1991. Common factors affecting bond returns. *Journal of Fixed Income*, **1**(1), 54–61.

Litterman, R., Scheinkman, J., & Weiss, L. 1991. Volatility and the yield curve. *Journal of Fixed Income*, **1**(49), 49–53.

Longstaff, Francis A, & Schwartz, Eduardo S. 1992. Interest Rate Volatility and the Term Structure: A Two-Factor General Equilibrium Model. *Journal of Finance*, **47**(4), 1259–82.

McAleer, M. 2009. The ten commandments for optimizing value-at-risk and daily capital charges. *Journal of Economic Surveys*, **23**(5), 831–849.

McCulloch, J. H. 1971. Measuring the term structure of interest rates. *The Journal of Business*, **44**(1), 19–31.

McCulloch, J. H. 1975. The tax-adjusted yield curve. *Journal of Finance*, **30**(3), 811–30.

Nelson, C. R. N., & Siegel, A. F. 1987. Parsimonious modeling of yield curves. *The Journal of Business*, **60**(4), 473–489.

Nelson, D.B. 1991. Conditional heteroskedasticity in asset returns: a new approach. *Econometrica*, **59**(2), 347–370.

Poon, S.H., & Granger, C.W.J. 2003. Forecasting Volatility in Financial Markets: A Review. *Journal of Economic Literature*, **41**(2), 478–539.

Rezende, R. B., & Ferreira, M. S. 2011. Modeling and forecasting the yield curve by an extended Nelson-Siegel class of models: A quantile autoregression approach. *Journal of Forecasting*, **30**(8), 339–350.

Robinson, P.M., & Zaffaroni, P. 2006. Pseudo-maximum likelihood estimation of ARCH(∞) models. *The Annals of Statistics*, **34**(3), 1049–1074.

Santos, A. A. P., & Moura, G. V. 2012. Dynamic factor multivariate GARCH model. *Forthcoming, Computational Statistics and Data Analysis.*

Santos, A. A. P., Nogales, F.J., & Ruiz, E. 2012a. Comparing univariate and multivariate models to forecast portfolio value-at-risk. *Forthcoming, Journal of Financial Econometrics.*

Santos, A. A. P., Ruiz, E., Nogales, F.J., & Van Dijk, D. 2012b. Optimal portfolios with minimum capital requirements. *Journal of Banking and Finance*, **36**, 1928–1942.

Sheppard, K. 2003. Multi-step estimation of multivariate GARCH models. *In: Proceedings of the International ICSC. Symposium: Advanced Computing in Financial Markets.*

Svensson, L. O. 1994 (Sept.). *Estimating and interpreting forward interest rates: Sweden 1992-1994.* IMF Working Papers 94/114. International Monetary Fund.

Vlaar, P.J.G. 2000. Value at risk models for Dutch bond portfolios. *Journal of banking & finance*, **24**(7), 1131–1154.

Zaffaroni, P. 2007. Contemporaneous aggregation of GARCH processes. *Journal of Time Series Analysis*, **28**(4), 521–544.

Zakoian, J.M. 1994. Threshold heteroskedastic models. *Journal of Economic Dynamics and control*, **18**(5), 931–955.

Zivot, E. 2009. Practical issues in the analysis of univariate GARCH models. *In:* Andersen, T.G., Davis, R.A., Kreiss, J.-P., & Mikosch, T. (eds), *Handbook of Financial Time Series.* Springer Verlag.

Appendix: Univariate GARCH models considered

In this appendix we describe the univariate GARCH specifications that were used to model the conditional variance of the factors and the conditional variance of the measurement errors.

GARCH:

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Glosten-Jagannathan-Runkle GARCH (GJR-GARCH):

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \gamma I[\epsilon_{t-1} < 0]\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

Exponential GARCH (EGARCH):

$$\ln(\sigma_t^2) = \omega + \alpha\frac{|\epsilon_{t-1}|}{\sqrt{\sigma_{t-1}^2}} + \gamma\frac{\epsilon_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \beta\sigma_{t-1}^2$$

Threshold GARCH (TGARCH):

$$\sigma_t = \omega + \alpha|\epsilon_{t-1}| + \gamma I[\epsilon_{t-1} < 0]|\epsilon_{t-1}| + \beta\sigma_{t-1}$$

Asymmetric power GARCH (APARCH):

$$\sigma_t^\lambda = \omega + \alpha\left(|\epsilon_{t-1}| + \gamma\epsilon_{t-1}\right)^\lambda + \beta\sigma_{t-1}^\lambda$$

Asymmetric GARCH (AGARCH):

$$\sigma_t^2 = \omega + \alpha(\epsilon_{t-1} + \gamma)^2 + \beta\sigma_{t-1}^2$$

Nonlinear asymmetric GARCH (NAGARCH):

$$\sigma_t^2 = \omega + \alpha(\epsilon_{t-1} + \gamma\sqrt{\sigma_{t-1}^2})^2 + \beta\sigma_{t-1}^2$$