INVESTMENT GRADE COUNTRIES YIELD CURVE DYNAMICS

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Abstract

The U.S. market has dominated the empirical on fixed income, but the globalization process has brought a new dimension up in the world's bond market. Currently the U.S. bonds correspond to less than 50% of all private and governmental bonds issued around the world. At the same time, the sovereign bonds of emerging markets have increased steadily since their debts has been renegotiated. In search for superior returns and with a long run horizon for investments, pension funds and mutual investment funds has absorbed these bonds. However, in spite of such a growing importance, emerging markets bonds studies have been neglected by the literature. Then, this work comes to bridge such gap and uses Diebold, Li and Yue's (2006) framework to extract global factors related to sovereign bonds from investment grade emerging markets. The possibility that emerging markets react to global factors is great, but it is neither clear nor well established by any other paper to the best of our knowledge. Our contribution is to determine whether there exist such global factors, how they explain the term structure dynamics of each country, and compare the results with other existing studies. Our results indicate the emerging markets global factors are very connected to developed market factors, both qualitatively and in magnitudes of parameters.

JEL Classification: C32, C51, F37, G12

Key-words: Sovereign bonds, emerging markets, yield curve, global fator

1 Introduction

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The term structure of interest rate stands for the relationship between bond yields and their redemption date, or maturity. Macroeconomic variables and other latent factors determine such relationship and therefore have been studied for a considerable time by academics. The term structure analysis provides a method to extract information from the interaction between those variables, and to forecast how changes in the economic environment may affect the shape of the term structure.

The focus of most studies about yield curve has been on the single-country case using idiosyncratic macroeconomic factors to model the yield. For example, Ang and Piazzesi (2003) use affine models to explain the term structure of the interest rates.

The assumption that latent factors generates the yield curve has widely driven the literature on term structure, and some instances are Litterman and Scheinkman (1991); Balduzzi, Das, and Sundaram (1996); Bliss (1997a); and Dai and Singleton (2000). Such factors are usually interpreted as level, slope and curvature, according to Andersen and Lund (1997), Diebold and Li (2006), and Diebold, Rudebusch and Aruoba (2006).

Because interactions in the global bond markets are very complex, the need to study the relationship in a cross-country environment is enormous. Notwithstanding, it is uncommon to focus on cross-country market, except Diebold, Li and Yue (2006), who determine the existence of common global yield factors. Specifically, they show the dynamics of cross-country bond interactions using a few countries from OCDE. Their paper follows Diebold and Li (2006), an extension of Nelson and Siegel's (1987) framework, in a way that the model is a hierarchical dynamic setting for the countries' yield curve, which depend both on idiosyncratic factors and global factors. Despite the fact the model follows a different framework from others that are in the global environment, such as Solnik (1974) and Thomas and Wickens (1993), it shares similar concerns. Then, Diebold, Li and Yue (2006) determine how the factors are interrelated between each other.

Another important concern is how to measure the global factors. Observed macroeconomic global factors are inadequate to explain the yield curve because each country's macroeconomics measurement methodology may be quite different. On the other hand, any attempt to extract those variables using, for example, principal components analysis is potentially inferior than more structured methodologies that take into account latent variables as the Kalman Filter. Hence, the correct measurement of existing, but latent, global factors is crucial to quantify country vulnerability or market integration.

The paper is organized as follows. Section 2 presents the theoretical model. Section 3 extends the model to include global factors in emerging markets. Section 4 discusses the main results, and Section 5 concludes.

2 Theoretical Model

2.1 The Nelson-Siegel's and Diebold-Li's Models

Milton Friedman's claim regarding to the need of a parsimonious model to describe the yield curve has inspired Nelson and Siegel (1987), henceforth NS, to first propose a model for describing that curve. Friedman says: "Students of statistical demand functions might find it more productive to examine how the whole term structure of yield can be described more compactly by a few parameters." Then, NS follows Friedman's advice and come up with a simple and parsimonious model, but sufficiently flexible to represent the most common shapes associated with the yield curve: monotonically increasing, humped, and S shaped.

Typical yield curve shapes are generated by a class of functions associated with the solutions of differential and difference equations. For instance, let $d_t(m)$ denote the price of an $m - periods$ discounted bound, i.e, $d_t(m)$ is the present value at time t of \$1 receivable m periods from today. Let $y_t(m)$ denote the continuously compounded zero-coupon nominal yield to maturity, or spot rate. From the yield curve it is possible to obtain the discount curve:

$$d_t(m) = e^{-m \times y_t(m)} \tag{1}$$

The continuously compounded spot rate is the single rate of return applied until the maturity of m years from today:

$$y_t(m) = -\frac{\ln(d(m))}{m}$$

Another important concept is the *forward rate,* f_t, which measures the prevalent rate in each point in the future. The forward rate average defines the yield to maturity as follows:

$$y_t(m) = \frac{1}{m} \times \int_0^m f_t(x)dx, \tag{2}$$

where $f_t(x)$ denotes the forward rate curve as a function of the maturitie m and $t = 1, 2, ..., T$.

Hence, from the discount curve (1) and (2) it is possible to obtain the instantaneous (nominal) forward rate curve:

$$f_t(m) = -\frac{d_t'(m)}{d_t(m)} = -\left[y_t(m) + m \times y_t'(m)\right]. \tag{3}$$

The heuristic motivation to investigate how the shapes are comes from the expectation theory on the term structure of interest rates. If the spot rates are generated as solutions of a differential equation, then the solution of this equation will be also a forward rate. For instance, NS considered a second order differential equation to describe the movements of the yield curve, and hence, with the assumption of real and unequal roots, the solution will be the forward rate:

$$f_t(m) = b_{0,t} + b_{1,t} \times e^{-\lambda_{1,t} m} + b_{2,t} \times e^{-\lambda_{2,t} m}, \tag{4}$$

where

$\lambda_{1,t}$ and $\lambda_{2,t}$ are time factor loadings associated with the equations;

$b_{0,t}$, $b_{1,t}$, and $b_{2,t}$ are coefficients to be determined based on initial conditions; and $t = 1, ..., T$.

Hence, the equation (4) gives us a family of forward rate curves whose shapes depend on the values of $b_{1,t}$, and $b_{2,t}$, while $b_{0,t}$ is the asymptote.

There are some problems associated with that function. Depending on the parameters λ_1 and λ_2, there is more than one value for bs that generate similar curves, so as the bs are not unique. Another problem appears when the convergence is not achived after performing the nonlinear estimation, what suggests that the function is overparameterized.

In order to overcome such difficulties, NS suggested a more parsimonious model. It generates the same range of shapes of previous specification, but differs from equation (4) by having identical roots as equation (5) shows:

$$f_t(m) = b_{0,t} + b_{1,t} \times e^{-\lambda_t m} + b_{2,t} \times \left(\lambda_t m \times e^{-\lambda_t m}\right). \tag{5}$$

The model may be viewed as a constant plus a Laguerre function, that is a polynomial times an exponential decay term, which belongs to a mathematical class of approximating functions[1]. Then, the solution for the yield as a function of maturity may be found by solving equation (2):

$$y_t(m) = b_{0,t} + (b_{1,t} + b_{2,t}) \times \frac{\left(1 - e^{-\lambda_t m}\right)}{\lambda_t m} - b_{2,t} \times e^{-\lambda_t m}. \tag{6}$$

for $t = 1, 2, ..., T$.

The limiting path of $y(m)$, when m increases, is its asymptote $b_{0,t}$; and, when m is small, the limit is $(b_{0,t} + b_{1,t})$. Different shapes can be drawn by varying the parameters λ and bs. If $\lambda_t = 1$, $b_{0,t} = 1$,$(b_{0,t} + b_{1,t}) = 0$, and $b_{2,t} = a$ where $a \in \mathbb{Z}$, then equation (6) becomes:

$$y_t(m) = 1 - \frac{(1-a) \times (1 - e^{-m})}{m} - a \times e^{-m}$$

Considering a given period t and varying the parameter a between -6 and 12, representing the curves from below to above, the figure shows the possible shapes that one can draw:

[1]Please, see details in Abramowitz, Milton and Irene (1965, ch. 22), and Whittaker and Watson (1990).

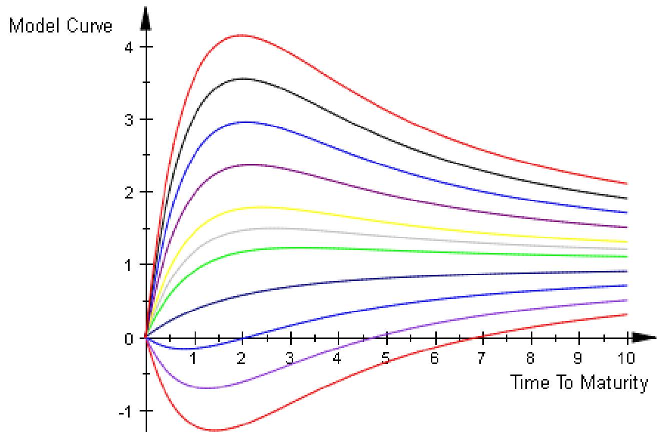


Figure 1: Yield Curve Shapes

NS's model creates two shortcomings. The first is conceptual and claims that it is difficult to give intuitive interpretations for the factors. The second is operational and says it is hard to estimate precisely the factors, because some multicolinearity can emerge. Then, Diebold and Li's (2006), henceforth DL, proposes another factorization:

$$y_t(m) = \beta_{0,t} + \beta_{1,t} \times \frac{\left(1 - e^{-\lambda_t m}\right)}{\lambda_t m} + \beta_{2,t} \times \left[\frac{\left(1 - e^{-\lambda_t m}\right)}{\lambda_t m} - e^{-\lambda_t m}\right]. \tag{7}$$

The main distinction is the way they factorize equation (6). The original NS model matches the DL's when $b_{0,t} = \beta_{0,t}$, $b_{1,t} = \beta_{1,t} + \beta_{2,t}$, $b_{2,t} = \beta_{2,t}$. DL's factorization is preferable to NS's because both $\frac{\left(1-e^{-\lambda_t m}\right)}{\lambda_t m}$ and $e^{-\lambda_t m}$ have similar decreasing shapes, and, if $b_{1,t}$ and $b_{2,t}$ are interpreted as factors, their respective loadings, $\frac{\left(1-e^{-\lambda_t m}\right)}{\lambda_t m}$ and $e^{-\lambda_t m}$, would be very similar.

The factor loadings 1, $\frac{\left(1-e^{-\lambda_t m}\right)}{\lambda_t m}$ and $e^{-\lambda_t m}$, can be easily extracted using the maturities m and a specific constant λ. They can be interpreted as measuring the strength of long, medium and short term components of the forward rate or of the yield curve.

The parameter λ is related to the exponential decay rate. Small values of λ produce slow decay, and fit well long term maturities; by contrast, large values of λ produce fast decay and fit better curves that have short term maturities. DL choose a constant value for λ, in such a way to maximize the curvature loading. Since, it is usually observed that maturity finds the maximum point between 2 and 3 years[2], DL use the average between these two maturities and set $\lambda = 0.0609$, corresponding to 30 months.

The shape of the factor loadings, 1, $\frac{(1-e^{-\lambda m})}{\lambda m}$, and $\left[\frac{(1-e^{-\lambda m})}{\lambda m} - e^{-\lambda m}\right]$ are illustrated in figure (2). As mentioned before, the factor corresponding to $\beta_{0,t}$ represents the long term, the one corresponding $\beta_{2,t}$ represents the medium term, and the last one corrsponding to $\beta_{1,t}$ represents the short term.

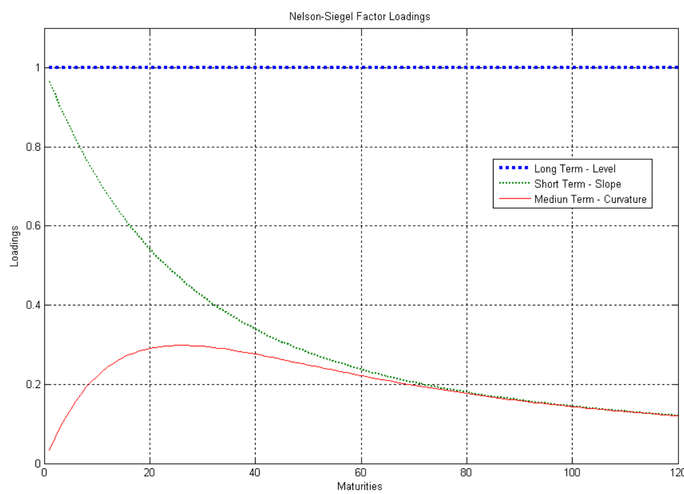


Figure 2: Nelson-Siegel Factor Loadings

[2] See that in figure 2.

3 The Global Model

Last section showed that studies of the U.S. closed-economy environment using a generalized DL model fit well the dynamics of the yield curve. Now, this section extends the basic model to a multi-country environment, following Diebold, Li and Yue (2006), henceforth DLY.

Using the DL factorization of the NS yield curve for a single country and indexing the parameters to represent a specific country, the model is:

$$y_{i,t}(m) = l_{i,t} + s_{i,t} \times \frac{\left(1 - e^{-\lambda_{i,t}m}\right)}{\lambda_{i,t}m} + c_{i,t} \times \left[\frac{\left(1 - e^{-\lambda_{i,t}m}\right)}{\lambda_{i,t}m} - e^{-\lambda_{i,t}m}\right] + \varepsilon_{i,t}(m), \quad (8)$$

where

$y_{i,t}(m)$ is the continously-compounded zero-coupon nominal yield of a bond maturing m periods ahead in country i at period t;

$i = 1, 2, \ldots, N$, and $t = 1, 2, \ldots, T$;

$\varepsilon_{i,t}(m)$ represents a disturbance with variance $\sigma_i^2(m)$.

The coefficients are interpreted as latent factors. They are the level, the slope, and the curvature, denoted, respectively, by l, s, and c.

DL simplifies the model in equation (9). The first simplification makes $\lambda_{i,t}$ constant across countries and time for the same reason as before. The authors argue there is a tiny loss of generality from doing that, since λ determines the maturity at which the curvature loading reaches the maximum. The second simplification makes $c_{i,t} = 0$ for all t and i. The argument for doing that comes from the fact that missing data makes the estimated curve be considerably imprecise at very short and/or very long maturities. They also allege that the curvature is not associated with macroreconomic fundamentals, as level is connected to inflation and slope is connected with GDP or capacity of utilization. Hence, the model can be written as:

$$y_{i,t}(m) = l_{i,t} + s_{i,t} \times \frac{\left(1 - e^{-\lambda m}\right)}{\lambda m} + \varepsilon_{i,t}(m). \quad (9)$$

In Diebold, Rudebusch and Aruoba (2006), the single-country version was expressed in terms of a space-state framework, such that equation (9) represents the space equation, and the time varying parameters l_{it} and s_{it}, which follow a first-order diagonal autoregression vector, represents the state equations.

From the single-country model, one may adapt it to an N-country approach, coupled with a similar space-state framework. The problem now is that the global yield to maturity $Y_t(m)$ is not observed as well as the factors, that is:

$$Y_t(m) = L_t + S_t \times \frac{\left(1 - e^{-\lambda m}\right)}{\lambda m} + v_t(m) \tag{10}$$

where

$Y_t(m)$ is the theoretical global yield;

L_t is the global level; and

S_t is the global slope.

These latent global factors are common to every country. It is postulated that the global yield factors follow a first-order VAR model as follows:

$$\begin{pmatrix} L_t \\ S_t \end{pmatrix} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \begin{pmatrix} L_{t-1} \\ S_{t-1} \end{pmatrix} + \begin{pmatrix} U_t^l \\ U_t^s \end{pmatrix} \tag{11}$$

where

U_t^n is the structural disturbance $n = l, s$;

$E\left(U_t^n\right) = 0$; and

$E\left[U_t^n U_{t'}^{n'}\right] = \begin{cases} (\sigma^n)^2, \text{ if } t = t' \text{ and } n = n' \\ 0, \text{ otherwise} \end{cases}.$

Then the model decomposes the country-specific level (slope) into a global level (slope) and some idiosyncratic fator, $\varepsilon_{i,t}^n$, whose mean is null:

$$l_{i,t} = \alpha_i^l + \beta_i^l L_t + \varepsilon_{i,t}^l, \tag{12a}$$

$$s_{i,t} = \alpha_i^s + \beta_i^s S_t + \varepsilon_{i,t}^s, \tag{12b}$$

for every $i = 1, 2, \ldots, N$.

It is assumed that the country idiosyncratic factors also follow an $AR(1)$ model:

$$\begin{pmatrix} \varepsilon_{i,t}^l \\ \varepsilon_{i,t}^s \end{pmatrix} = \begin{pmatrix} \theta_{i,11} & \theta_{i,12} \\ \theta_{i,21} & \theta_{i,22} \end{pmatrix} \begin{pmatrix} \varepsilon_{i,t-1}^l \\ \varepsilon_{i,t-1}^s \end{pmatrix} + \begin{pmatrix} u_{i,t}^l \\ u_{i,t}^s \end{pmatrix} \tag{13}$$

where

$u_{i,t}^n$ is a disturbance;

$E\left[u_{i,t}^n\right] = 0$; and

$E\left[u_{i,t}^n u_{i',t'}^{n'}\right] = \begin{cases} (\sigma_i^n)^2, \text{ if } t = t', i = i', \text{ and } n = n'; \\ 0, \text{ otherwise.} \end{cases}$

In addition we assume that $E\left[u_{i,t-s}^n U_t^{n'}\right] = 0$ for all n, n', i and s.

In terms of the state-space model, the equations (11) and (13) are transition equations. We can represent them more compactly using matrix notation as follows:

$$\begin{bmatrix} y_{1,t}(m_1) \\ y_{1,t}(m_2) \\ \vdots \\ y_{N,t}(m_{J-1}) \\ y_{N,t}(m_J) \end{bmatrix}_{JN\times 1} = A \begin{bmatrix} \alpha_1^l + \varepsilon_{1,t}^l \\ \alpha_1^s + \varepsilon_{1,t}^s \\ \vdots \\ \alpha_N^l + \varepsilon_{N,t}^l \\ \alpha_N^s + \varepsilon_{N,t}^s \end{bmatrix} + B \begin{bmatrix} L_t \\ S_t \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t}(m_1) \\ \epsilon_{1,t}(m_2) \\ \vdots \\ \epsilon_{N,t}(m_{J-1}) \\ \epsilon_{N,t}(m_J) \end{bmatrix}, \tag{14a}$$

where

N is the number of countries;

J is the number of maturities;

A and B are conforming matrices:

$$A = \begin{pmatrix} 1 & \frac{1-e^{-m_1\lambda}}{m_1\lambda} & 0 & \cdots & 0 & 0 \\ 1 & \frac{1-e^{-m_2\lambda}}{m_2\lambda} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & 1 & \frac{1-e^{-m_{J-1}\lambda}}{m_{J-1}\lambda} \\ 0 & 0 & \cdots & \cdots & 1 & \frac{1-e^{-m_J\lambda}}{m_J\lambda} \end{pmatrix}_{JN\times 2J}, \tag{14b}$$

and

$$B = \begin{pmatrix} \beta_1^l & \beta_1^s\left(\frac{1-e^{-m_1\lambda}}{m_1\lambda}\right) \\ \beta_1^l & \beta_1^s\left(\frac{1-e^{-m_2\lambda}}{m_2\lambda}\right) \\ \vdots & \vdots \\ \beta_N^l & \beta_N^s\left(\frac{1-e^{-m_{J-1}\lambda}}{m_{J-1}\lambda}\right) \\ \beta_N^l & \beta_N^s\left(\frac{1-e^{-m_J\lambda}}{m_J\lambda}\right) \end{pmatrix}_{JN\times 2}. \tag{14c}$$

The global common factor, $\beta's$ and the factor loading, $\left(\frac{1-e^{-m_j\lambda}}{m_j\lambda}\right)$ are not separately identified in equation (14c). Because of this, we assume that β_M^l is positive and are able to identify the signs of factors and factors loadings. Moreover, because of the magnitudes of global factors and factor loadings, we consider the innovations to global factors and factor loadings have unit standard deviation, that is, $\sigma^n = 1$, $n = l, s$[3].

[3]Sargent and Sims (1977) and Stock and Watson (1989) have proposed such identification restriction.

3.1 Econometric Strategy

The estimation method in a multi-country enviroment can be done using equation (14a). The state-space can be estimated by Kalman Filter, and fully-efficient Gaussian maximum likelihood dynamics estimates obtain. In the single-country case, estimating the latent factors using Kalman Filter is relatively easy, because the number of parameters is small. In the multi-country case, however, one-step maximum likelihood is difficult to implement, due to the large number of parameters to estimate. Hence, DLY propose a convenient multi-step estimation method. The first step is to obtain the latent factors (level and slope) for each country. The second step consists of taking the estimates previously obtained and use them in equations (11), (12a), (12b) and (13) to extract the global factors. We describe that strategy in what follows.

3.1.1 Estimating the Countries' Level and Slope

In equation (6) one sees that λ varies over time, following the original NS model. It is possible to obtain the parameters $l_{i,t}$, $s_{i,t}$ and λ_t by nonlinear cross-sectional least squares at each period t. To make things easier, we follow DL and fix $\lambda = 0.0609$, the point where the curvature is maximum. Then we compute the factor loadings for each maturity[4] and estimate the parameters $l_{i,t}$ and $s_{i,t}$ by ordinary least squares, for each country i and period t. Hence, there are two estimated parameters in each month for each country[5].

3.1.2 Estimating the Global Factors

The estimation of the Global Factors is made by Kalman Filter. However, the methodology is very sensitive to the initial values. Therefore, there are a series of intermediate steps before the final estimation. These steps are supposed to generate appropriate initial values for the Kalman Filter. In order to accomplish this aim, we start by using Principal Component Analysis on the level, $l_{i,t}$, and the slope, $s_{i,t}$, to extract a proxy for the global level factor (L_t^{PCA}) and global slope factor (S_t^{PCA}). Then, we estimate the following $VAR(1)$:

$$\begin{pmatrix} L_t^{PCA} \\ S_t^{PCA} \end{pmatrix} = \begin{pmatrix} \phi_{11}^{PCA} & \phi_{12}^{PCA} \\ \phi_{21}^{PCA} & \phi_{22}^{PCA} \end{pmatrix} \begin{pmatrix} L_{t-1}^{PCA} \\ S_{t-1}^{PCA} \end{pmatrix} + \begin{pmatrix} Z_t^l \\ Z_t^s \end{pmatrix},$$

[4] The factor loadings are not stochastic. They are 1 and $\frac{1-e^{-m_j\lambda}}{m_j\lambda}$.

[5] In the appendix we describe another way of extracting such factors. We have estimated the model using both techniques and there was no important change.

and use the estimated coefficientes $\phi_{ii}^{PCA}, i = 1, 2$, to initialize the Kalman Filter.

Also, it is necessary to obtain the initial values for the idiosyncratic factors for each country. In order to do that, make the following regression for $i = 1, 2, \ldots, N$:

$$\begin{aligned} l_{i,t} &= c_i^l + \delta_i^l \times L_t^{PCA} + \varepsilon_{i,t}^{PCA,l} \\ s_{i,t} &= c_i^s + \delta_i^s \times S_t^{PCA} + \varepsilon_{i,t}^{PCA,s} \end{aligned} \tag{15}$$

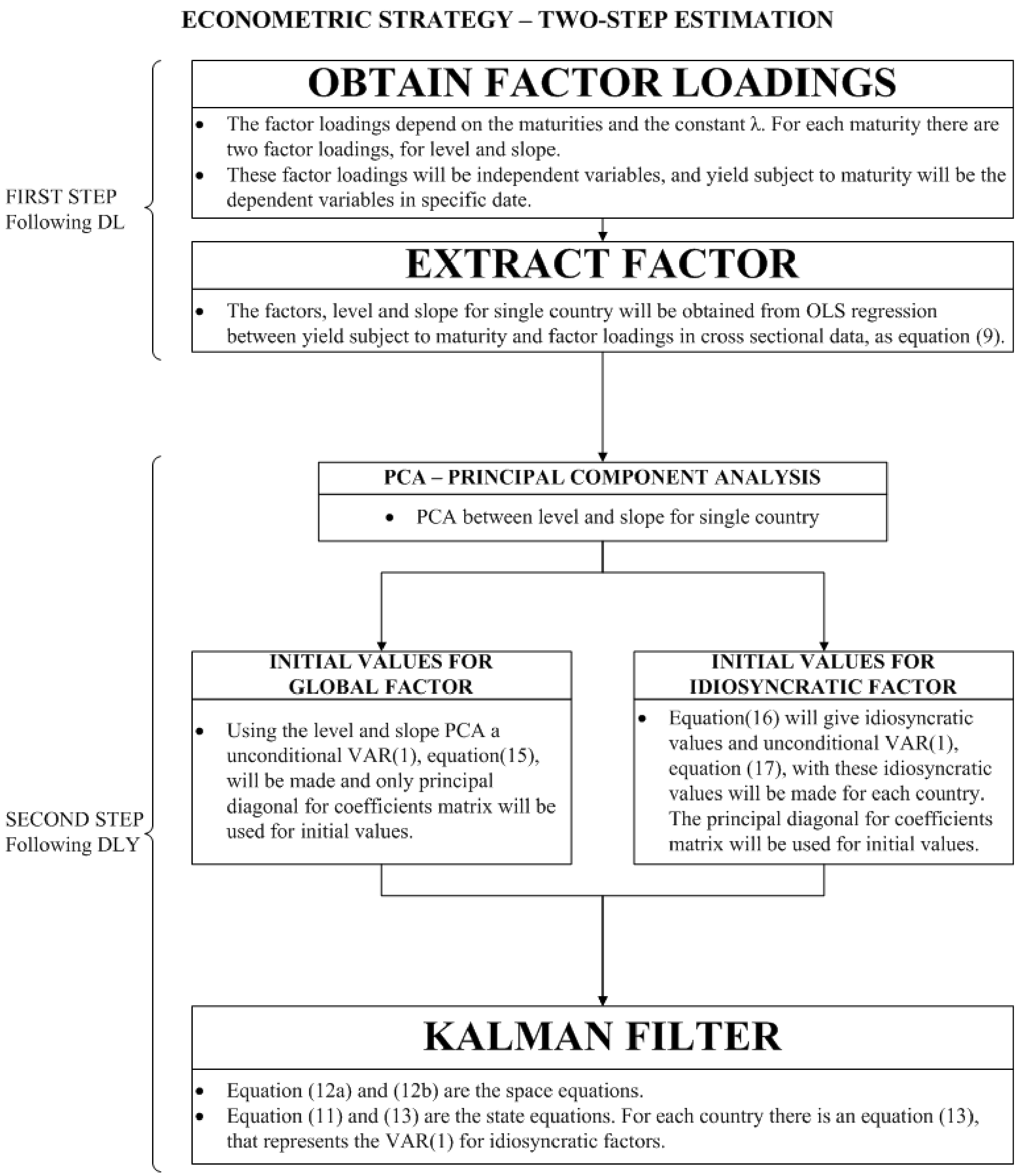
Then, the series $\varepsilon_{i,t}^{PCA,l}$ and $\varepsilon_{i,t}^{PCA,s}$ are collected, and a new $VAR(1)$ is estimated for each country:

$$\begin{pmatrix} \varepsilon_{i,t}^{PCA,l} \\ \varepsilon_{i,t}^{PCA,s} \end{pmatrix} = \begin{pmatrix} \theta_{i,11}^{PCA} & \theta_{i,12}^{PCA} \\ \theta_{i,21}^{PCA} & \theta_{i,22}^{PCA} \end{pmatrix} \begin{pmatrix} \varepsilon_{i,t-1}^{PCA,l} \\ \varepsilon_{i,t-1}^{PCA,s} \end{pmatrix} + \begin{pmatrix} r_{i,t}^l \\ r_{i,t}^s \end{pmatrix}$$

Because, there is no evidence that the errors are correlated, only the parameters $\theta_{i,11}^{PCA}$and $\theta_{i,22}^{PCA}$ for $i = 1, 2, \ldots, N$ will be used as initial values.

In short, the equations (11), (12a),(12b) and (13) follow a space-state system and will be estimated by Kalman Filter. The equations (12a) and (12b) are the measurements, and the autoregressive vector equations (11) and (13) are the state equations.

The numbers of coefficients estimated is $2 + 8N$. In the measurement equation, there are $4N$ parameters to be estimated (β_i^l, β_i^s, α_i^l , α_i^s), four for each country. In the state equations, there are $2 + 2N$ parameteres to be estimated, two parameters relative to the global factors ($\phi_{1,1}$, $\phi_{2,2}$) and two parameters for the idiosyncratic factors for each country ($\theta_{i,1,1}, \theta_{i,22}$). The standard-deviation are considered constant over time. For idiosyncratic the factors there are two standard-deviation for each country.



Note - This chart demonstrate the econometric strategy

Figure 3: Econometric Strategy Chart

4 Empirical Results

4.1 Data Analysis

4.1.1 Construction

The data[6] consist of government zero-coupon bond yields. The bonds are in terms of US currency and are from South Africa, South Korea, and Mexico. Mexico is a Latin American and underdeveloped country, however highly connected with the US. Korea experiences a very different environment compared to South Africa. A common point about these countries is the fact they have faced with economic crises in the last few years and they recently were considered investment grade for agencies of risk. The aim of this paper is to check whether they share common characteristics and comparem them with developed countries.

The data is taken from the last working day of each month and span from June 1998 to September 2007. They have different maturities in each country. Thus, the maturities are interpolated in order to synchronize maturity dates and the number of bonds of each country.[7] The interpolation method is the cubic spline, following sugestions of McCulloch (1971,1975).

4.1.2 Description

The maturities correspond to 3, 6, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69 and 72 months. The following table show the basic statistics for some maturities.

[6]We would like to thank Fernando Siqueira to provide us the data.
The Data ware obtained in Bloomberg Broadcast.

[7]More information is in the Appendix.

Maturity	Mean	Std. Deviation	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{\rho}(30)$	Max.	Min.
South Africa							
3	10.15	3.39	0.938	0.243	-0.032	23.49	6.60
6	10.40	3.26	0.941	0.264	-0.030	23.94	6.72
12	10.50	3.01	0.940	0.336	-0.019	22.56	7.05
24	10.90	3.17	0.950	0.422	0.000	22.06	7.16
48	11.38	4.59	0.949	0.453	0.024	21.97	7.26
60	11.59	5.49	0.954	0.483	0.029	21.16	7.32
72	11.79	6.47	0.955	0.501	0.027	20.90	7.40
South Korea							
3	4.84	2.57	0.958	0.514	-0.045	13.20	1.16
6	5.01	2.63	0.957	0.502	-0.045	13.82	1.29
12	5.25	2.56	0.957	0.510	-0.050	13.93	1.59
24	5.58	2.44	0.953	0.512	-0.056	14.06	2.38
48	6.22	2.26	0.946	0.510	-0.033	14.45	3.40
60	6.34	2.21	0.942	0.490	-0.028	14.65	3.22
72	6.43	2.18	0.942	0.482	-0.027	14.79	3.70
Mexico							
3	4.28	2.19	0.913	0.487	-0.070	13.30	1.07
6	4.47	2.22	0.910	0.498	-0.083	13.95	1.12
12	4, 75	2.19	0.914	0.533	-0.097	14.17	1.25
24	5.29	2.12	0.910	0.567	-0.108	14.39	1.50
36	5.89	2.02	0.903	0.562	-0.072	14.48	2.46
48	6.39	1.98	0.907	0.609	-0.040	14.41	3.40
60	6.83	1.97	0.917	0.602	-0.022	14.19	4.13
72	7.093	1.90	0.930	0.604	-0.001	13.31	4.63

Maturity in months

Table 1: Summary Statistics of Bond Yields

The South Africa's averages have the highest interest rates. The countries have upward-slope yield curves. The volatility tends to decrease as long as maturities increase, in which case there is no clear tendency. All yields are highly persistent for all countries, and the persistence tend to increase for South Africa and Mexico.

The figures (4), (5) and (6) show the government bond yield curves across countries and time. The countries yield curves tend to be similar and moderately homogeneous. In the begining of the sample there is the effect of Russia's crises and afterwards the curves tend to be smoother.

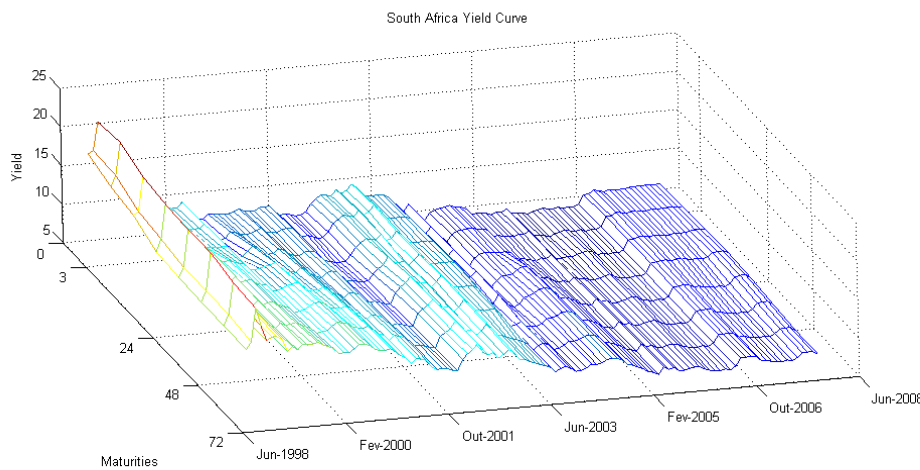


Figure 4: Yield Curve across Maturity and Time

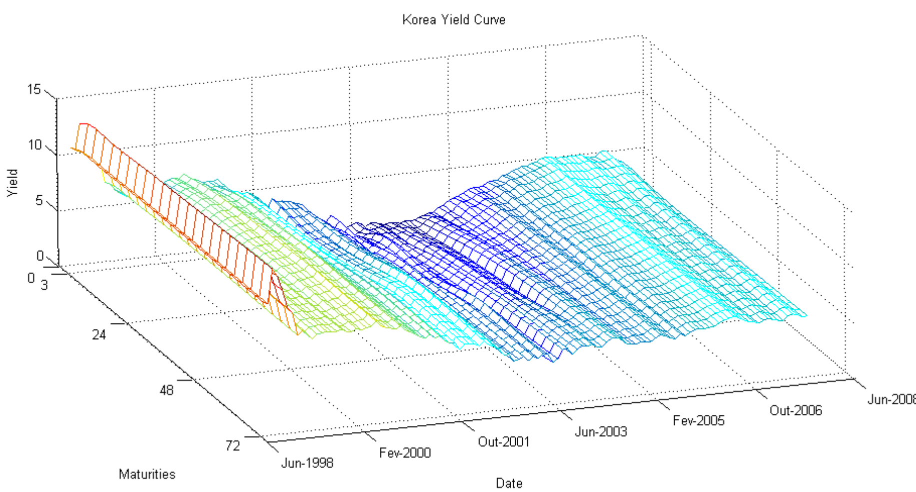


Figure 5: Yield Curve across Maturity and Time

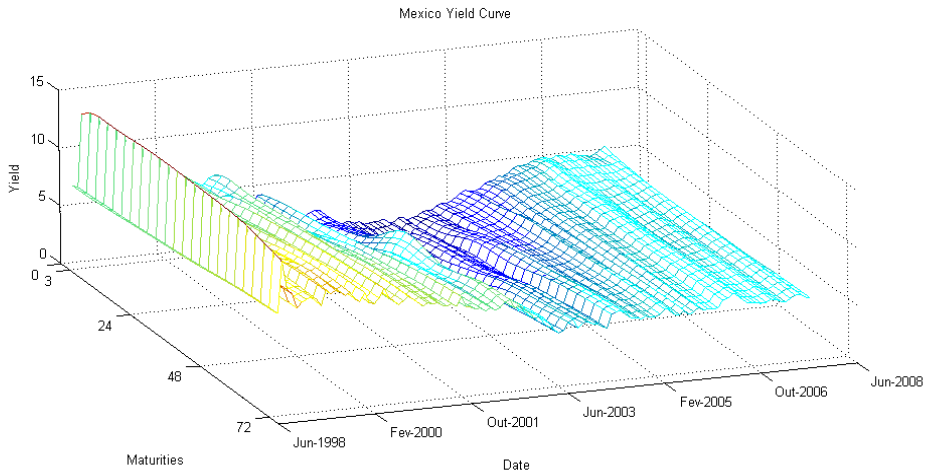


Figure 6: Yield Curve across Maturity and Time

4.1.3 Preliminary Analysis

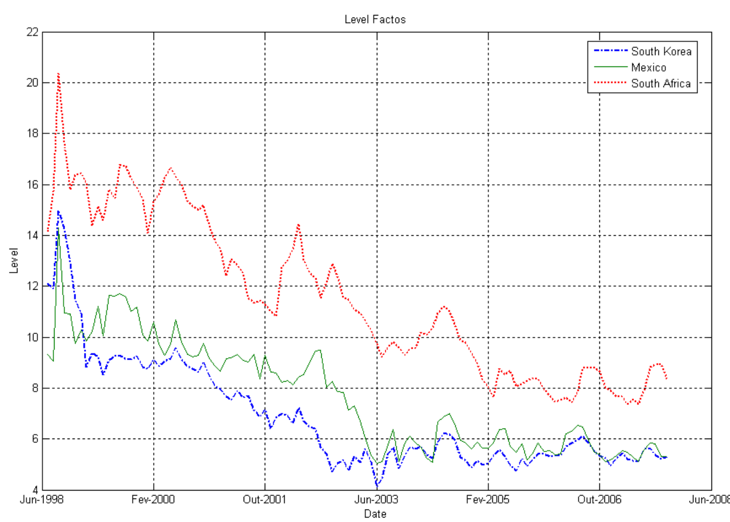
The multi-step methodology generates a number of series and estimates to be analyzed. Many of these estimates carries features to be also found by the end of the procedure. In the first step, we have extracted estimates of the level and the slope, l_{it} and s_{it}, for each t and i, using DL's two step method.

The descriptive statistic of the factors so estimated indicates similar pattern across countries. The level is always positive and the slope is always negative. The first autocorrelation for the level and slope in all countries is persistent around 0.95 and 0.91. The Ng-Perron unit root test using Bartlet kernel method with lag length 12 without tendency did not reject the existence of unit root in both factors in all countries. However, it should be less than one, because otherwise the nominal bond yield would eventually become negative, which is not acceptable. DLY find similar results in their work.

South Africa								
Factor	Mean	Std. Dev.	Max.	Min.	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{\rho}(30)$	MZa
$\widehat{l}_{i,t}$	11.53	3.13	20.36	7.35	0.95	0.510	0.035	-1.05
$\widehat{s}_{i,t}$	-1.42	2.47	4.83	-7.26	0.91	-0.024	-0.140	-1.53
South Korea								
Factor	Mean	Std. Dev.	Max.	Min.	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{\rho}(30)$	MZa
$\widehat{l}_{i,t}$	6.86	2.17	15.00	4.12	0.93	0.47	-0.01	0.33
$\widehat{s}_{i,t}$	-2.37	1.70	0.58	-5.70	0.94	0.26	0.09	-2.17
Mexico								
Factor	Mean	Std. Dev.	Max.	Min.	$\widehat{\rho}(1)$	$\widehat{\rho}(12)$	$\widehat{\rho}(30)$	MZa
$\widehat{l}_{i,t}$	7.65	2.14	14.29	5.04	0.93	0.54	0.03	$-2,00$
$\widehat{s}_{i,t}$	-4.22	2.58	0.25	-8.77	0.91	0.21	0.09	-4.53

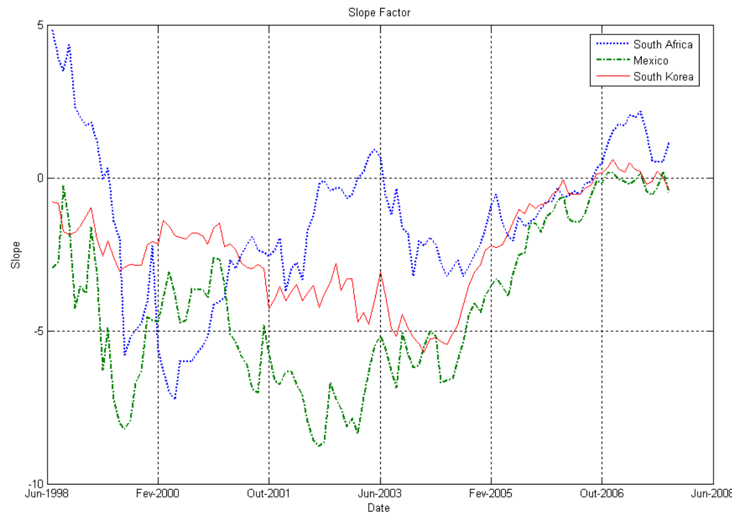
MZa Denotes an argumented Ng-Perron test Statistic

Table 2: Descriptive Statistics for the Estimated Factors



OLS estimation

Figure 7: Level Factors using DL



OLS estimation

Figure 8: Slope Factor using DL

An insight comes up from analyzing the figures: both the level and slope are converging for the same level. Although one might say that Korea and Mexico were always linked, careful analysis shows the convergence tendency only has consolidated after 2005. South Africa always had some link with the other countries, but in the level factor South Africa curve is at a higher level. For instance, during international crises like Russia's crises in 1998, both level and slope are connected. We suggest that in normal periods, the curves are tied in with each other and after 2005 such connection was reinforced. Other insight from the figures show that the level, which is associated with inflation, and slope, associated with GDP growth, are stabilizing around the same level.

From these curves, one can extract the global fators and compare the results with DLY's for developed countries. It is possible to notice that the oscillations in level and slope factor in developed markets is smoother than in emerging markets.

4.2 Main Results

The previous figures made clear that, despite the variability, the dynamics of the factors run in the same directions. That evidence is better described by using the principal components analysis, PCA, which indeed indicate the existence of a global level and a slope factor.

The first principal component for level explains more than 90% of variation, and more than 68% for the slope. The following tables bring details about the procedure.

	Comp 1	Comp 2	Comp 3
Eigenvalue	17.648	0.833	0.426
Variance Prop.	0.933	0.044	0.023
Cumulative Prop.	0.933	0.977	1.000

Table 3: Principal Components Analysis for the Estimated Level Factor

	Comp 1	Comp 2	Comp 3
Eigenvalue	10.543	4.287	0.583
Variance Prop.	0.684	0.278	0.037
Cumulative Prop.	0.684	0.962	1.000

Table 4: Principal Components Analysis for the Estimated Slope Factor

Although the PCA method provides a way of extracting the commom factor across the series, the Kalman Filter makes the best prediction and therefore should bem more precise[8]. The initial parameters were calculated earlier. Then, we impose a VAR(1) model for estimating simultaneously the global factors and country and idyossincratic coefficients, presented in the next table.

The analysis shows that the global yield factors are highly serially correlated, consistent with DLY found. All country level factors load positively and significantly on the respective global yield factor. The effect, however, is greater in South Africa, perhaps because South Africa achieved the economic stability after others. The influence in the level factor is 0.52 for South Africa, while it is 0.40 and 0.65 for Korea and Mexico, respectively.

Concerning the slope, the impact is negative and significant. The country idiosyncratic factor, like a global factor, is also highly correlated. The global and idiosyncratic parameters fail to be significant in the intercept of all slopes.

[8]Maximization made by using Marquart algorithm with convergence criteria of 0.0001.

Global Level Factor	Global Slope Factor
$L_t = \underset{(0.05)}{0.95^*} L_{t-1} + U_t^l$	$S_t = \underset{(0.03)}{0.98^*} S_{t-1} + U_t^s$
Country Level Factor	
$l_{A,t} = \underset{(2.72)}{11.45^*} + \underset{(0.11)}{0.52^*} L_t + \varepsilon_{A,t}^l$	$\varepsilon_{A,t}^l = \underset{(0.05)}{0.93^*} \varepsilon_{A,t-1}^l - \underset{(0.16)}{0.97^*} v_{A,t}^l$
$l_{K,t} = \underset{(3.41)}{7.86^{**}} + \underset{(0.08)}{0.40^*} L_t + \varepsilon_{K,t}^l$	$\varepsilon_{K,t}^l = \underset{(0.03)}{0.98^*} \varepsilon_{K,t-1}^l - \underset{(0.15)}{2.03^*} v_{K,t}^l$
$l_{M,t} = \underset{(3.00)}{7.52^*} + \underset{(0.07)}{0.65^*} L_t + \varepsilon_{M,t}^l$	$\varepsilon_{M,t}^l = \underset{(0.28)}{-0.59^{**}} \varepsilon_{M,t-1}^l - \underset{(0.73)}{3.70^*} v_{M,t}^l$
Country Slope Factor	
$s_{A,t} = \underset{(3.31)}{-0.07} - \underset{(0.16)}{0.19^*} S_t + \varepsilon_{A,t}^s$	$\varepsilon_{A,t}^s = \underset{(0.06)}{-0.95^*} \varepsilon_{A,t-1}^s - \underset{(0.10)}{050^*} v_{A,t}^s$
$s_{K,t} = \underset{(2.89)}{-1.64} - \underset{(0.08)}{0.31^*} S_t + \varepsilon_{K,t}^s$	$\varepsilon_{K,t}^s = \underset{(0.09)}{0.90^*} \varepsilon_{K,t-1}^s - \underset{(0.56)}{2.45^*} v_{K,t}^s$
$s_{M,t} = \underset{(4.99)}{-3.06} - \underset{(0.13)}{0.51^*} S_t + \varepsilon_{M,t}^s$	$\varepsilon_{M,t}^s = \underset{(0.22)}{0.72^*} \varepsilon_{M,t-1}^s - \underset{(0.31)}{0.79^*} v_{M,t}^s$

Note-The numbers in the parentesis represents the standard Deviation , One asterisk Indicate that is significant at 1 percent, two asterisk Indicate that is significant at 5 percent

Table 5: Estimates of Global Yield Curve Models

Figure (9) plots the Global levels calculated by PCA and Kalman Filter for comparative purposes. The correlation between the series is nearly 97%.

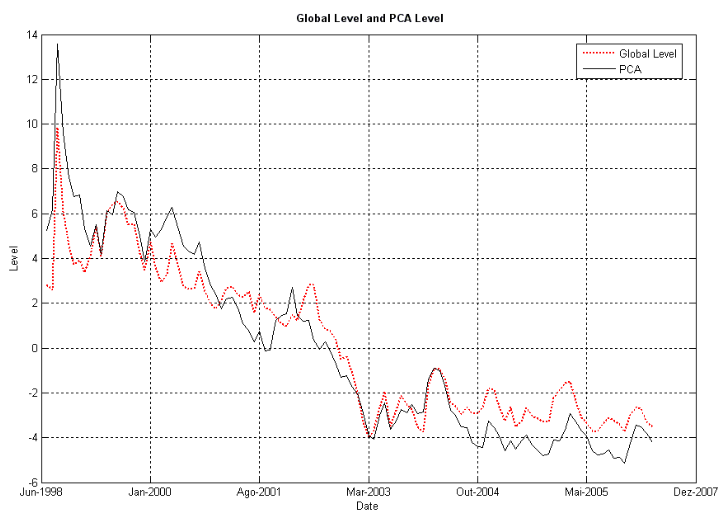


Figure 9: Global Level Factor vs. PCA Level

Comparing figure (9) with DLY, there is a similar pattern of having a negative level during the same period, meaning inflation is decreasing.

Figure (10) plots the Global slope calculated by PCA and Kalman Filter. The correlation is positive and is nearly 93%.

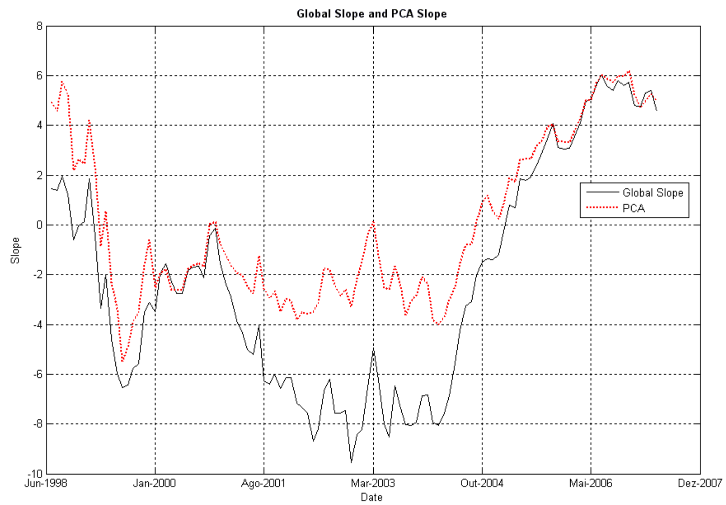


Figure 10: Global Slope Factor vs. PCA Slope

To our perception, Mexico seems to highly influence the Global Factors, mainly the level. The PCA procedure is very similar to Kalman filter, in particular with respect to the level, where it is possible to observe the same tendency and the same volatility.

The slope curve however is little affected by the global slope factor, since it fluctuates around zero, contrasting with DLY. A possible explanation is to realize that the level in our contries is higher than OECD's. Therefore, the slope factor might be described as some sort of transitory tendency on the yield curve. In developed countries, the yield level is lower than in emerging countries, so as changes in the emerging market level in the short run tend to be relatively higher than in developed countries. As a consequence, the slope in developed countries tend to capturate significantly such an increase in the volatility.

The discussion can be enriched if we make a connection between macroeconomic variables and latent factors. By associating level and slope to inflation and GDP growth of these three countries. Observe that in the periods without crises the movements of the curves tend to be smoother, while in the periods of crises, like that occurred in 1998 in Russia, the volatility increases. The stylized facts in the period after 2003 indicate that inflation has decreased, and GDP growth has been stable in those economies, while during the Russia's crises the inflation and GDP growth in those countries are more unstable.

4.3 Variance Decomposition

A specific country factor variance can be evaluated as a proportion the global's and idiosyncratic's variances. By doing that, one can explain the magnitude of variations of each factor and infer the influence of global movements in the country economy.

The formulation of country factor can be extracted from equations (12a) and (12b) using a simple definition of variance as follows:

$$\begin{array}{l} var(l_{i,t}) = \left(\beta_i^l\right)^2 \times var(L_t) + var(\varepsilon_{i,t}^l) \\ var(s_{i,t}) = \left(\beta_i^s\right)^2 \times var(S_t) + var(\varepsilon_{i,t}^s) \end{array}.$$

Although the global and idiosyncratic factors should not be correlated, the Kalman Filter procedure may not be able to make these factors fully orthogonal between each other, since it is a statistical procedure. Hence, it is wise to use another orthogonalization procedure. The OLS regression between the country level factor against the Global Factor makes the idyosincratic factor orthogonal to the Global factor.

$$\begin{array}{rcl} l_{i,t} & = & c_i^l + \delta_i^l \times L_t + \varepsilon_{i,t}^l \\ s_{i,t} & = & c_i^s + \delta_i^s \times S_t + \varepsilon_{i,t}^s \end{array}$$

Thus, the equations for variance decomposition becomes:

$$\begin{array}{l} var(l_{i,t}) = \left(\delta_i^l\right)^2 \times var(L_t) + var(\varepsilon_{i,t}^l) \\ var(s_{i,t}) = \left(\delta_i^s\right)^2 \times var(S_t) + var(\varepsilon_{i,t}^s) \end{array}.$$

Given such orthogonalization procedurem the variance decomposition is calculated and the results are:

Level Factors Volatility			
	South Africa	South Korea	Mexico
Global Factor	87.40%	74.31%	99.67%
Idiosyncratic Factor	12.60%	25.69%	0.33%
Slope Factors Volatility			
	South Africa	South Korea	Mexico
Global Factor	17.89%	91.04%	91.53%
Idiosyncratic Factor	82.11%	8.96%	8.47%

Table 6: Variance Decomposition

The influence of the global level factor in South Africa is 74.31%, in Mexico is about 99.67%, and about 87.40% in Korea. We argue that Mexico is closely related to the US economy, which in turn, drives the rest of world. Hence, a global curve from emerging countries is connected to the one from developed countries.

The global slope factor volatility affects more South Korea and Mexico because their greater connection between USA. Notwithstanding, the slopes are converging and the global slope is going to zero. Therefore, it is not suprising that such an evidence has come up.

5 Conclusion

The present work has extended the Diebold, Lee and Yue's (2006) model on global yield curves to cover emerging market countries. Issues of emerging markets bonds have increased considerably during last years. Therefore the U.S bonds have faced with riskier competitors that promise higher returns up to the point that emerging market bonds represent more than 50% of the transactions in that market. That makea the measurement of a common global factor for emerging markets very important for making decisions.

We found similar conclusions as DLY both qualitatively and sometimes in magnitude as well. Similar characteristics are found between South Korea, South Africa and Mexico's yield curve factors. Hence it affects considerably the extraction of the global factors, mainly the level. We argue that the large influence from the level global factor on these countries comes from external reasons. Identical conclusion emerges regarding the global slope factor.

As to numerical results, the global level factor influences Mexico's yield for level by a factor bigger than others. The negative trend of the global level shows why yields in emerging markets have fallen down. The fall in the global slope decreases yield variability; hence one can infer that if the slopes are getting stable in recent years, then the yields are varying less.

As to the influence of global and idiosyncratic factors on the level and slope variability of each country, notice that Mexico is the most affected by both, followed by Korea. The slope variability of each country indicates that Mexico and South Korea are more affected by the global slope factor than South Africa that is affected only in 17.89%. The results of global level and slope factor variability respect the stylized facts, since the emerging markets' yields are higher than in developed countries. Mexico should be most influenced by the U.S. bonds, and this must explain why the global factors estimated in this work affect it more than the other countries.

Thus we have extracted global factors from emerging markets yield curves. The results indicate the existence of a global level and global slope factors in the emerging markets. These factors are a very important and significant fraction for determining the bond yield behavior.

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APPENDIX A - Cubic Spline

The discount function gives the present value of \$1.00 to be received in m years. Hence, the correspondent yield to maturity of investment, $y(m)$, also known as *spot interest rate* or *zero coupon rates*, must satisfy the following equation under continuous compounding:

$$d(m)e^{my(m)} = 1 \Longrightarrow d(m) = e^{-my(m)}.$$

The definition of discount function and spline method can be expressed using k continuously differentiable functions $s_j(m)$ to approximate the discount rates:

$$y(m) = a_0 + \sum_{j=1}^{k} a_j s_j(m) \tag{16a}$$

or

$$d(m) = \exp\left[-m\left(a_0 + \sum_{j=1}^{k} a_j s_j\left(m\right)\right)\right] \tag{16b}$$

where

$s_j(m), j = 1, 2, \ldots, k$ are known functions with maturities m; and

$a_j, j = 0, 1, \ldots, k$ are unknown coefficients to be determined from the data.

Since the discount rate must satisfy the constraint $d(0) = 1$, set $a_0 = 1$ and $s_j(0) = 0$ for $j = 1, \ldots k$, and once the functional form of $s(m)$ is determined, the coefficients can be estimated by linear regression.

McCulloch (1971) used the quadratic spline, based on a quadratic polynomial of $s_j(m)$, and McCulloch (1977) used the cubic spline, based on a cubic polynomial. McCulloch's choice is due to easyiness of evaluating, differentiating, and integrating.

Bliss (1996) tested different methods to evaluate interpolations, and concludew that the unsmoothed Fama-Bliss (1987) method is better than others. However, the difference between Fama-Bliss' and the cubic spline is small, reason why we follows McCulloch's cubic spline in this work, based on the extension by Brennan and Xia (2003).

Consider the following points:

$$(m_0, Y_0), (m_1, Y_1), \ldots, (m_{n-1}, Y_{n-1}), (m_n, Y_n).$$

By using them, it is possible to fit a cubic spline through the data.

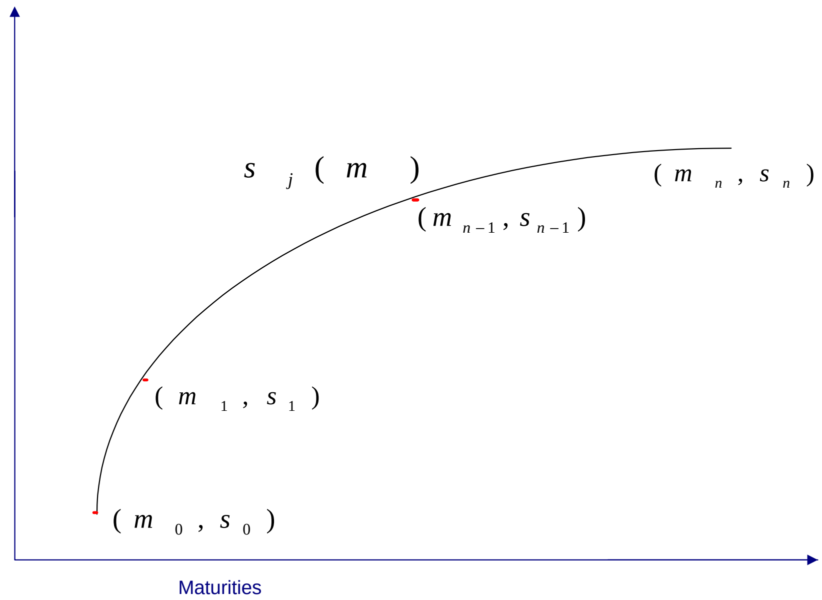

Figure 11: Interpolation of dicrete data

The splines are given by:

$$
\begin{aligned}
s_1(m) &= \varphi_1 m^3 + \rho_1 m^2 + \gamma_1 m + \zeta_1 \qquad m_0 \leq m \leq m_1 \\
s_2(m) &= \varphi_2 m^3 + \rho_2 m^2 + \gamma_2 m + \zeta_2 \qquad m_1 \leq m \leq m_2 \\
&: \\
&: \\
s_n(m) &= \varphi_n m^3 + \rho_n m^2 + \gamma_n m + \zeta_n \qquad m_{n-1} \leq x \leq m_n
\end{aligned}
$$

The $4n$ coefficients are obtained using simultaneous linear equations. The question that arises is whether the number of equations is the same as variables. In general, the number of equations is lower than the number of coefficientes. Then, to correct that, one can different the splines in different intervals that have the same central point. In cubic spline it must differentiate two times. Even though it must consider $\varphi_1 = 0$ or the first spline as a linear spline, to complete the numbers of equations.

Having these coefficients, one can use equation (16a) or (16b) and find the parameters φ.

APPENDIX B - Extracting the Factors by Kalman Filter

Diebold, Rudebush and Aruoba (2006) estimate the model by Kalman Filter. In their case, they make $l_{i,t}$ and $s_{i,t}$ as time varying coefficients and extract these latent factors from the data. First, assume that $l_{i,t}$ and $s_{i,t}$ follow a first order vector autoregression. Then, write the model in terms of a state-space system. The transition equation governs the dynamics of state vector[9], is:

$$\begin{pmatrix} l_{i,t} - \mu_{i,l} \\ s_{i,t} - \mu_{i,s} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} l_{i,t-1} - \mu_{i,l} \\ s_{i,t-1} - \mu_{i,s} \end{pmatrix} + \begin{pmatrix} n_{i,t}(l) \\ n_{i,t}(s) \end{pmatrix}, \tag{17}$$

where $\mu_{i,n}$ is the mean.

The observed system is given by:

$$\begin{pmatrix} y_{i,t}(m_1) \\ y_{i,t}(m_2) \\ \vdots \\ y_{i,t}(m_N) \end{pmatrix} = \begin{pmatrix} 1 & \frac{1-e^{-m_1\lambda}}{m_1\lambda} \\ 1 & \frac{1-e^{-m_2\lambda}}{m_2\lambda} \\ \vdots & \vdots \\ 1 & \frac{1-e^{-m_N\lambda}}{m_N\lambda} \end{pmatrix} \begin{pmatrix} l_{i,t} \\ s_{i,t} \end{pmatrix} + \begin{pmatrix} \epsilon_{i,t}(m_1) \\ \epsilon_{i,t}(m_2) \\ \vdots \\ \epsilon_{i,t}(m_N) \end{pmatrix}. \tag{18}$$

In the more compact matrix-notation, the state-space system is:

$$(F_{i,t} - \mu_i) = A(F_{i,t-1} - \mu_i) + \eta_{i,t}, \tag{19a}$$

where

$F_{i,t} \equiv \begin{pmatrix} l_{i,t} & s_{i,t} \end{pmatrix}'$;

$\mu_i \equiv \begin{pmatrix} \mu_{i,l} & \mu_{i,s} \end{pmatrix}'$;

$A \equiv \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$;

$\eta_{i,t} \equiv \begin{pmatrix} n_{i,t}(l) & n_{i,t}(s) \end{pmatrix}'$.

Equation 19a represents the transition equation. And the measurement equation is:

$$Y_{i,t} = \Lambda F_{i,t} + \epsilon_{i,t}, \tag{19b}$$

where

$Y_{i,t} \equiv \begin{pmatrix} y_{i,t}(m_1) & y_{i,t}(m_2) & \cdots & y_{i,t}(m_N) \end{pmatrix}'$;

[9] The initial values can be obtained by using the previous technique, i.e, Diebold and Li's (2006).

$$\Lambda \equiv \begin{pmatrix} 1 & 1 & \cdots & 1 \\ \frac{\left(1-e^{-\lambda m_1}\right)}{\lambda m_1} & \frac{\left(1-e^{-\lambda m_2}\right)}{\lambda m_2} & \cdots & \frac{\left(1-e^{-\lambda m_N}\right)}{\lambda m_N} \end{pmatrix};$$

$$\epsilon_{i,t} \equiv \begin{pmatrix} \epsilon_{i,t}(m_1) & \epsilon_{i,t}(m_2) & \cdots & \epsilon_{i,t}(m_N) \end{pmatrix}'.$$

The Kalman Filter requires that the white noise transition and measurement disturbance be orthogonal to each other and to initial states.

$$\begin{pmatrix} \eta_{i,t} \\ \epsilon_{i,t} \end{pmatrix} \sim WN\left[\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} Q & \mathbf{0} \\ \mathbf{0} & H \end{pmatrix}\right] \tag{20}$$

$$E(F_{i,t}\eta'_{i,t}) = \mathbf{0} \tag{21}$$

$$E(F_{i,t}\epsilon'_{i,t}) = \mathbf{0} \tag{22}$$

The analysis assumes that the matrix H is diagonal, which implies the standard result that yield maturities deviations from the yield curve are uncorrelated. The Q matrix may not be diagonal, in order to allow for correlated shocks from the latent factors.

The Kalman filter single country factor extraction provides similar results than DL method, and the differences are not more that 1%. Because of this, the level and slope will be extracted using DL.