

On the combination of multivariate volatility forecasts

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Very preliminary, comments welcome

Abstract

We devise a novel approach to combine predictions of high dimensional conditional covariance matrices using economic criteria based on portfolio selection. The combination scheme takes into account not only the portfolio objective function but also the portfolio characteristics in order to define the mixing weights. The proposed model combination i) does not require a proxy for the latent conditional covariance matrix, ii) does not require optimization of the combination weights, which facilitates its implementation in practical situation, and iii) holds the equally weighted model combination as a particular case. An empirical application involving 4 data sets with cross section dimensions ranging from 17 to 100 assets over a 10-year time span shows that the proposed performance-based combinations of multivariate GARCH forecasts leads to mean-variance portfolios with higher risk-adjusted performance in terms of Sharpe ratio as well as to minimum variance portfolios with lower risk on an out-of-sample basis with respect to all individual models considered. The results are robust to the presence of transaction costs.

Key words: Bootstrap, conditional correlation models, mean-variance portfolios, Sharpe ratios, transaction costs.

1 Introduction

Combining predictions from alternative models is a very well established forecasting approach. In fact, the literature on forecast combinations is extensive and points to the superiority of combined forecasts with respect to single models in many different contexts. The motivation to combine forecasts comes from an important result from the methodological literature on forecasting, which shows that a linear combination of two or more forecasts may yield more accurate predictions than using only a single forecast (Granger, 1989; Newbold and Harvey, 2002; Aiolfi and Timmermann, 2006). Moreover, adaptive strategies for

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combining forecasts might also mitigate structural breaks and model misspecification and thus lead to more accurate forecasts (Pesaran and Timmermann, 2007).

A number of potential explanations for the good performance of combined forecasts vis-a-vis individual forecast models can be pointed out. First, if two models provide partial, but incompletely overlapping explanations, then some combination of the two might do better than either alone; see Hendry and Clements (2004). Specifically, if two forecasts were differentially biased (one upwards, one downwards), then combining could be an improvement over either. Second, averaging forecasts reduces variance to the extent that separate sources of information are used. Third, forecast combination can also alleviate the problem of model uncertainty.¹

Existing evidence suggests that the combination of univariate volatility leads to more accurate forecasts with respect to single models (Becker and Clements, 2008; Patton and Sheppard, 2009). The study of combined multivariate volatility predictions, however, is in its early days. This issue is very important since the combination of multivariate volatility models might also lead to more accurate predictions, therefore improving decision making in economic and financial problems that depend on covariance estimates such as asset pricing, portfolio optimization and market risk management. Engle and Colacito (2006), for instance, show that accurate covariance information will allow the investor to achieve lower volatility, higher return, or both.

In this paper, we put forward a novel approach to combine multivariate volatility predictions from alternative conditional covariance models. The proposed combination rule is motivated by the fact this class of models is often applied in portfolio selection problems. More specifically, combination weights are defined by taking into account the past performance of each individual model in obtaining optimal portfolios. The past performance can be either in-sample or out-of-sample. In our empirical implementation of the proposed combination method, we rely on in-sample rather than out-of-sample portfolio performance of each individual model. This choice is motivated by two reasons. First, it simplifies implementation as the same estimation window used to estimate the parameters of each individual model is also used to evaluate portfolio performance. Second, and most importantly, we performed extensive robustness checks to evaluate the gains (or lack of) from changing to past out-of-sample portfolio performance, despite higher computational cost. The results are very similar in comparison to those based on the past in-sample performance of each individual model, which makes us confident to employ this approach. Another im-

¹Geweke and Amisano (2011) study the existence of model complementarities in linear prediction pools and show that a model with positive weight in a pool may have zero weight if some other models are deleted from that pool.

portant aspect of the proposed approach is that combination weights can be calibrated in order to adjust i) the aggressiveness in the allocation across alternative models (i.e. a cross section adjustment) and ii) the importance given to the most recent observations in the calculation of portfolio performance (that is, a time series adjustment). Three major advantages of this approach are that i) does not require a proxy for the latent conditional covariance matrix, ii) does not require optimization of the combination weights, and iii) holds the equally-weighted model combination as a particular case. Each of these advantages are discussed in Section 2.

The proposed approach to combine predictions of conditional covariance matrices is based on the mean-variance portfolio selection problem introduced by Markowitz (1952). Two aspects motivate our choice for adopting the mean-variance framework. First, economic applications involving multivariate volatility models often rely on the mean-variance portfolio problem; see, for example, Engle and Colacito (2006) and Engle and Sheppard (2008). Therefore, it is natural to consider this type of portfolio selection policy when devising a performance-based model combination approach for conditional covariance models. Second, the mean-variance framework is one of the milestones of modern finance theory and is widespread among academics and market participants. In this framework, individuals choose their allocations in risky assets based on the trade-off between expected return and risk. We consider two alternative versions of the mean-variance portfolio problem. The first is based on an investor who wishes to minimize portfolio risk subjected to a target portfolio return. In this case, the investor wishes to achieve higher risk-adjusted portfolio returns. The mean-variance problem, however, is known to be very sensitive to estimation of the mean returns (e.g. Jagannathan and Ma, 2003). Very often, the estimation error in the mean returns degrade the overall portfolio performance and introduces an undesirable level of portfolio turnover. In fact, existing evidence suggest that the performance of optimal portfolios that do not rely on estimated mean returns is better; see, for instance, DeMiguel *et al.* (2009). Because of that, we also consider a second type of investor who adopts the minimum variance criterion in order to decide her portfolio allocations. This portfolio policy can be seen as a particular case of the traditional mean-variance optimization. In fact, existing evidence suggest that the performance of optimal portfolios that do not rely on estimated mean returns is better; see, for instance, DeMiguel *et al.* (2009).

It is worth noting that both mean-variance and minimum variance portfolio problems require estimation of the full covariance matrix of asset returns. In most practical situations, however, the investor faces uncertainty on which is the most appropriate specification to model and to forecast covariances. For

instance, should the investor employ a multivariate GARCH model or instead a multivariate stochastic variance (SV) specification? Should the investor consider time varying or time invariant conditional correlations? Are more parameterized specifications better than more parsimonious ones? How to handle this uncertainty? All these questions are typically answered in the literature by horse-racing many single models; see, for instance, Engle and Colacito (2006) and Engle and Sheppard (2008). Our performance-based combination approach helps alleviating this problem since it is explicitly built to exploit model complementarities and to favor the models that generate portfolios with better portfolio performance and to penalize those that yield portfolios with poor performance.

Our paper is related to recent studies such as Becker *et al.* (2014) and Amendola and Storti (2015). Becker *et al.* (2014) study the ability of alternative loss functions to select the best specifications for the portfolio selection problem and find that a likelihood-based function is the best function for this purpose. Amendola and Storti (2015) pioneered the literature on combined multivariate volatility predictions and extended the approach in Patton and Sheppard (2009) to a multivariate setting. Their approach involves estimating model combination weights via minimization of a loss function and similar as in Becker *et al.* (2014) requires a proxy for the unobserved latent covariance matrix, which is specified as a realized covariance measure. Our approach, in contrast, differs in several aspects since does not require neither optimization of combination weights nor a proxy for the unobserved covariance matrix, which greatly facilitates its implementation in practical situations. Moreover, instead of adopting a pure statistical criterion for estimating the mixing weights, we directly apply an economic criterion for defining, in a dynamic fashion, how much weight to put in each model. Therefore, our approach to combine multivariate volatility predictions is consistent with the fact that this class of models is ultimately applied to economic problems.

We test the effectiveness of the proposed model combination using 4 data sets from the US, European and Asian stock markets with cross section dimensions ranging from 17 to 100 assets over a 10-year time span. We implement a set of 8 alternative multivariate GARCH models that are widely used in portfolio selection problems and obtain model combination schemes using the mean-variance and minimum variance portfolio criterion for the mixing. We conduct a detailed economic evaluation of the resulting optimal portfolios and implement a test for the portfolio risk and for the portfolio Sharpe ratio based on the bootstrap procedure of Politis and Romano (1994), which allows us to formally compare optimal portfolios obtained with alternative specifications in terms of their sample characteristics. Our results

can be summarized as follows. First, when combining multivariate volatility forecasts based on the mean-variance criterion, we observe that the resulting conditional covariance estimators yield mean-variance portfolios with higher risk-adjusted returns measured by the Sharpe ratio. Second, when combining multivariate volatility forecasts based on the minimum variance criterion, we observe that the minimum variance portfolios obtained with the combined estimators are substantially and statistically less risky with respect to all individual models considered.

The rest of the paper is organized as follows. In section 2 we detail the proposed combination rule. Section 3 provide an empirical application based on two alternative data sets with many assets. Finally, section 4 concludes.

2 Combining multivariate volatility forecasts

In this section, we detail the proposed approach to combine multivariate volatility predictions in a dynamic fashion. To solve a variety of important problem in finance, it is common to choose a conditional covariance specification in order to account for the time variation in second order moments; see, for instance, Engle and Colacito (2006) and Becker *et al.* (2014). In most practical situations, however, the volatility process is subject to changes and the investor faces uncertainty on which is the most appropriate specification to model and to forecast covariances at a given point in time. In order to identify the best specification, the most common practice in the literature is to horse-race many single models. We, on the other hand, consider the possibility of improving the portfolio performance via model combinations. More specifically, we consider the case in which there exists M alternative candidates. In this case, the combined conditional covariance estimator, H_t^{Comb} , is defined as

$$H_t^{Comb} = \lambda_{1,t}H_t^1 + \dots + \lambda_{M,t}H_t^M, \quad (1)$$

where H_t^m denotes conditional covariance matrix of the m -th candidate model and $\lambda_{m,t}$ is the corresponding combination weight such that $\sum_{m=1}^M \lambda_{m,t} = 1$ and $\lambda_{m,t} \geq 0 \forall m$. It is worth noting that a linear combination in (1) is conveniently chosen as it guarantees that the resulting H_t^{Comb} is positive-definite provided that individual models also deliver positive-definite conditional covariance matrices.

The most important aspect when combining alternative model based forecasts is to specify a combination vector $\lambda_t = \{\lambda_{1,t}, \dots, \lambda_{M,t}\}$, that is, to decide how much weight to put in each individual forecast.

The literature on forecast combinations offers many options to specify λ_m . In the case of volatility models, for instance, Patton and Sheppard (2009) and Amendola and Storti (2015) consider a combination scheme based on the minimization of a statistical loss function that requires a proxy for the true unobserved volatility, which is specified as a realized volatility measure. Despite the fact that the realized covariance matrix is only an estimate of the true volatility, higher frequency data to compute this measure is not always available and the combination weights do not incorporate any information about the decision making task in which forecasts will be ultimately used.

We take a different approach with respect to the existing literature and specify λ_m in (1) by taking into account an economic criterion based on the mean-variance and minimum variance portfolio optimization problems. Using these economic criteria has at least two appealing features. First, it is consistent with the fact that multivariate volatility models are ultimately applied to economic problems. Second, it is based on a very well established portfolio selection problem. Next we detail the economic gain functions used to construct the combined conditional covariance estimator in (1).

2.1 Mean-variance model combination

We consider an investor who allocates her wealth in N alternative risky assets. In order to choose the weights w_i for $i = 1, \dots, N$ of each asset in the portfolio, we assume the investor adopts the mean-variance portfolio policy. In this setting, the investor wishes to minimize portfolio risk subjected to a target portfolio return. This portfolio optimization problem is defined as

$$\begin{aligned} & \min_{w \in \mathbb{R}^N} w_t' H_t^m w_t \\ & \text{subject to} \\ & w_t' \mu = \mu_0 \\ & \sum_{i=1}^N w_i = 1, \end{aligned} \tag{2}$$

where w_t is the vector of portfolio weights for time t chosen at time $t - 1$ and H_t^m is a positive-definite conditional covariance matrix of asset returns obtained with the m -th candidate model for time t and forecasted at time $t - 1$. μ is the vector of expected returns and μ_0 is the required return. In our empirical implementation, we define μ_0 as the return of the equally-weighted portfolio. The solution to (2) is given by $w_t = (H_t^m)^{-1} \frac{\mu(C\mu_0 - B) + \iota(A - B\mu_0)}{AC - B^2}$, where $A = \mu(H_t^m)^{-1}\mu$, $B = \mu(H_t^m)^{-1}\iota$, $C = \iota'(H_t^m)^{-1}\iota$, and ι is a vector of ones; see Cochrane (2009).

Upon solving the portfolio problem in (2) for a sample with T observations and adopting a given specification for the conditional covariance matrix H_t^m , the investor is interested in computing the portfolio risk-adjusted return, which is usually measured by the Sharpe ratio and is defined as $SR = \hat{\mu}/\hat{\sigma}$ where $\hat{\mu} = \frac{1}{T-1} \sum_{t=1}^{T-1} w_t' R_{t+1}$ is the average portfolio return and $\hat{\sigma}^2 = \frac{\sum_{t=1}^{T-1} (w_t' R_{t+1} - \hat{\mu})^2}{T-1}$, where R_{t+1} is the vector of asset returns at time $t + 1$. Instead of computing both $\hat{\mu}$ and $\hat{\sigma}_m^2$ as the standard sample mean and sample variance, respectively, we follow Stock and Watson (2004) and Genre *et al.* (2013) and consider a more general expression for the model's performance metric by introducing a discount (or forgetting) factor δ , i.e.

$$\begin{aligned}\hat{\mu}_m &= \frac{\sum_{t=1}^{T-1} \delta^{T-1-t} (w_t' R_{t+1})}{T-1} \\ \hat{\sigma}_m^2 &= \frac{\sum_{t=1}^{T-1} \delta^{T-1-t} (w_t' R_{t+1} - \hat{\mu})^2}{T-1}.\end{aligned}\tag{3}$$

Values of δ which are below unity assign higher weights to more recent portfolio variances in the calculation of combination weights. In our implementation, we follow Genre *et al.* (2013) and set $\delta = \{1, 0.95, 0.85, 0.80, 0.75\}$. The case where $\delta = 1$ corresponds to no discounting and is equivalent to the common expression of the sample variance.

In the mean-variance context, the combined conditional covariance estimator in (1) should privilege the models that yield higher risk-adjusted returns (i.e. higher Sharpe ratios) and penalize those that yield lower risk-adjusted returns. In this case, the performance-based model combination vector λ_m can be defined as

$$\lambda_m = \frac{\left(\hat{\mu}_m/\hat{\sigma}_m\right)}{\sum_{m=1}^M \left(\hat{\mu}_m/\hat{\sigma}_m\right)},\tag{4}$$

where $\hat{\mu}_m$ and $\hat{\sigma}_m^2$ are, respectively, the δ -adjusted portfolio average return and portfolio variance defined in (3). In order to rule out the possibility of selecting models that generate portfolios with negative SR, one can set $\lambda_m = 0$ if $SR_m < 0$. We refer to this approach as the mean-var(δ) combination rule. The inspiration for defining the performance-based model combination in (4) comes from Kirby and Ostdiek (2012), who devised a portfolio policy that takes into account the inverse of the sample variance of the individual assets as well as their risk-adjusted performance.

It is also worth noting that the mean-var(δ) strategy in (4) and (3) belongs to a more general class of

mean-var combination with weights of the form

$$\lambda_m = \frac{\left(\hat{\mu}_m/\hat{\sigma}_m\right)^\eta}{\sum_{m=1}^M \left(\hat{\mu}_m/\hat{\sigma}_m\right)^\eta}. \quad (5)$$

The idea behind this generalization is straightforward. The tuning parameter $\eta \geq 0$ determines how aggressively we adjust the mixing weights in response to changes in the realized portfolio Sharpe ratios obtained with each of the candidate models. As $\eta \rightarrow 0$ we recover the equally weighted model combination, and as $\eta \rightarrow \infty$ the weight on the model that yields the highest realized Sharpe ratio approaches 1. In our implementation, we follow Kirby and Ostdiek (2012) and consider alternative values of η such that $\eta = \{1, 2, 4, 10\}$. We refer to the mixing in (5) as the mean-var(δ, η) combination rule. Finally, substituting equation (5) into (1) yields a more general expression for the mean-var(δ, η) combined estimator:

$$H_t^{Comb} = \frac{1}{\sum_{m=1}^M \left(\hat{\mu}_m/\hat{\sigma}_m\right)^\eta} \sum_{m=1}^M \left(\hat{\mu}_m/\hat{\sigma}_m\right)^\eta H_t^m. \quad (6)$$

2.2 Minimum variance model combination

We now consider an alternative performance-based model combination rule based on the minimum variance portfolio policy. A very large body of literature in portfolio optimization considers this particular policy; see, for instance, Clarke *et al.* (2006, 2011) for extensive practitioner-oriented studies on the performance and on the composition of minimum variance portfolios. This policy can be seen as a particular case of the traditional mean-variance optimization. The mean-variance problem, however, is known to be very sensitive to estimation of the mean returns (e.g. Jagannathan and Ma, 2003). Very often, the estimation error in the mean returns degrade the overall portfolio performance and introduces an undesirable level of portfolio turnover. In fact, existing evidence suggest that the performance of optimal portfolios that do not rely on estimated mean returns is better; see, for instance, DeMiguel *et al.* (2009).

The minimum variance portfolio problem is defined as

$$\begin{aligned} & \min_{w \in \mathfrak{R}^N} w_t' H_t^m w_t \\ & \text{subject to} \\ & \sum_{i=1}^N w_i = 1, \end{aligned} \quad (7)$$

where w_t is the vector of portfolio weights for time t chosen at time $t - 1$ and H_t^m is a positive-definite conditional covariance matrix of asset returns obtained with the m -th candidate model for time t and forecasted at time $t - 1$. The solution to (7) is defined as $w_t = \iota'(H_t^m)^{-1}/\iota'(H_t^m)^{-1}\iota$, where ι is a vector of ones.

Upon solving the portfolio problem in (7) for a sample with T observations and adopting a given specification for the conditional covariance matrix H_t^m , the investor computes the δ -adjusted portfolio variance as defined in (3). In the minimum variance context, the combined conditional covariance estimator in (1) should privilege the models that yield lower portfolio risk (i.e. lower portfolio variance) and penalize those that yield higher portfolio risk. In this case, after accounting for the time series adjustment by means of the δ parameter as well as for the cross section adjustment by means of the η parameter, the performance-based minimum variance model combination vector λ_m can be defined as

$$\lambda_m = \frac{\left(1/\hat{\sigma}_m^2\right)^\eta}{\sum_{m=1}^M \left(1/\hat{\sigma}_m^2\right)^\eta}, \quad m = 1, \dots, M. \quad (8)$$

We refer to the mixing in (8) as the min-var(δ, η) combination rule. Finally, substituting equation (8) into (1) yields a more general expression for the min-var(δ, η) combined estimator:

$$H_t^{Comb} = \frac{1}{\sum_{m=1}^M \left(1/\hat{\sigma}_m^2\right)^\eta} \sum_{m=1}^M \left[\left(1/\hat{\sigma}_m^2\right)^\eta H_t^m\right]. \quad (9)$$

2.3 Important remarks

The combined conditional covariance estimators in (6) and in (9) use the past performance of each individual model in delivering portfolios with higher risk-adjusted performance and lower portfolio risk, respectively, in order to define the combination weights.² The aggressiveness in the allocation across alternative models is calibrated by the tuning parameter η . Moreover, the importance given to the most recent observations in the calculation of portfolio variances is calibrated by the discount factor δ . Therefore, while the η parameter performs a cross section adjustment on the aggressiveness in the allocation across alternative models, the discount factor δ performs a time series adjustment as it controls for the importance of most recent observations.

²Alternative algorithms to combine predictions for the conditional mean based on the past performance of the candidate models are also proposed in Yang (2004) and Hibon and Evgeniou (2005).

It is also worth emphasizing four important aspects of the model combination in (6) and (9). First, the mixing does not require a proxy for the latent conditional covariance matrix. This proxy is usually defined by means of a realized covariance matrix based on high-frequency data as in Laurent *et al.* (2012), Laurent *et al.* (2013), and Becker *et al.* (2014). The proposed approach, on the other hand, dispenses the use of intraday data and can be implemented with data sampled at any frequency. Second, the mixing rule does not require optimization of the combination weights, which facilitates its implementation in practice.

Finally, the fourth important aspect of the of the proposed approach to combine multivariate volatility predictions is that it holds the equally weighted model combination as a particular case (when $\eta = 0$). This specific model combination is found to outperform more sophisticated combination schemes in many contexts; see, for instance, Clemen (1989), De Menezes *et al.* (2000), Wallis (2011), and Genre *et al.* (2013). Some authors have argued this result is due to the instability of combination weights, which can deteriorate the performance of optimal combinations; see Kang (1986). This instability has its roots in the sampling error, which contaminates the estimated weights and is exacerbated by the collinearity that typically exists among primary forecasts (Diebold, 1989). The imposition of equal weights eliminates variation in the estimated weights and increases robustness with respect to model uncertainty, time-variation of parameters, and estimation errors that arise when combination weights have to be estimated (Palm and Zellner, 1992). Finally, Armstrong (2001) and Timmermann (2006) establish conditions under which adopting the equally weighted model combination is optimal. Timmermann (2006), for instance, argues that equal weights are optimal in situations with an arbitrary number of forecasts when the individual forecast errors have the same variance and identical pairwise correlations. These arguments motivate us to choose the equally weighted model combination $\text{min-var}(\eta = 0)$ as the main benchmark among all individual and combined covariance specifications considered in the paper.

2.4 Individual candidate models

In order to implement the model combination rules defined in (6) and (9), it necessary to implement a set of individual candidate models. For that purpose, we assume that the multivariate system of asset returns is modeled as $R_t = z_t(H_t^m)^{1/2}$. To model H_t^m , it is possible to consider a broad set of conditional covariance specifications including multivariate GARCH and SV models; see Asai *et al.* (2006), Bauwens *et al.* (2006), Silvennoinen and Teräsvirta (2009), and Chib *et al.* (2009). In this paper, we follow Engle

and Sheppard (2008) and Becker *et al.* (2014) and restrict our attention to the former class and implement a set of $M = 8$ alternative multivariate GARCH specifications widely used in portfolio selection problems:

Exponentially weighted moving average (EWMA): The EWMA model is defined as

$$H_t = \alpha R_{t-1}' R_{t-1} + (1 - \alpha) H_{t-1},$$

where α is a nonnegative parameter. When the α is set to a fixed value of 0.04, the EWMA is equivalent to the popular RiskmetricsTM approach. Zaffaroni (2008) shows that although it permits sizable computational gains and provide a simple way to impose positive semi-definitiveness of the resulting conditional covariance matrices, the RiskmetricsTM delivers non-consistent parameter estimates. Therefore, in our implementation of the EWMA specification the parameter α is estimated via maximum likelihood; see details below.

Optimal rolling estimator (ORE): The general rolling estimator is defined as $H_t = \sum_{k=1}^{\infty} \Omega_{t-k} \odot R_{t-k}' R_{t-k}$, where Ω_{t-k} is a symmetric matrix of weights and \odot denotes the element-by-element multiplication. The optimal weighting scheme proposed in of Foster and Nelson (1996) is given by $\Omega_{t-k} = \alpha \exp(-\alpha k) \iota \iota'$ where ι is a vector of ones. Therefore, the general rolling estimator can be rewritten as

$$H_t = \alpha \exp(-\alpha) R_{t-1}' R_{t-1} + \exp(-\alpha) H_{t-1},$$

where α is a nonnegative parameter. The ORE specification has been applied in many portfolio selection problems such as in Fleming *et al.* (2001, 2003) and Pooter *et al.* (2008). Fleming *et al.* (2003), in particular, point out that covariance matrix forecasts based on the ORE specification results in better portfolios in comparison to those obtained with other (unrestricted) multivariate GARCH models. The authors argue that the smoothness of the rolling estimator as the main reason for this.

Scalar VECH: The scalar VECH specification of Bollerslev *et al.* (1988) is defined as

$$H_t = C' C + \alpha R_{t-1} R_{t-1}' + \beta H_{t-1}.$$

Instead of estimating $N(N + 1)/2$ unique elements of C , we employ the variance targeting technique as suggested in Engle and Mezrich (1996). The general idea is to estimate the intercept matrix by an auxiliary estimator that is given by $\hat{C}'\hat{C} = \bar{S}(1 - \alpha - \beta)$, where $\bar{S} = \frac{1}{T} \sum_{t=1}^T R_t R_t'$, thus yielding the variance-targeting scalar VECH model

$$H_t = \bar{S}(1 - \alpha - \beta) + \alpha R_{t-1} R_{t-1}' + \beta H_{t-1},$$

which is covariance-stationary provided that $\alpha + \beta < 1$.

Orthogonal GARCH (O-GARCH): The O-GARCH model of Alexander (2001) belongs to a class of factor models and is able to achieve significant computational gains via dimensionality reduction. The O-GARCH model is given by $\Sigma_t = W\Omega_t W$, where W is a $N \times k$ matrix whose columns are given by the first k eigenvectors of the $t \times N$ matrix of asset returns, and Ω_t is a $k \times k$ diagonal matrix whose elements are given by $h_{f_{kt}}$ where $h_{f_{kt}}$ is the conditional variance of the k -th principal component and follows a GARCH(1,1) process. We implement the O-GARCH model using 3 principal components.

Conditional correlation models: This class of models is defined as $H_t = D_t \Psi_t D_t$, where D_t is a $N \times N$ diagonal matrix with diagonal elements given by $h_{i,t}$, where $h_{i,t}$ is the conditional variance of the i -th asset and follows a GARCH(1,1) process, and Ψ_t is a symmetric conditional correlation matrix with elements $\rho_{ij,t}$, where $\rho_{ii,t} = 1$, $i, j = 1, \dots, N$. We consider 4 alternative specifications to model Ψ_t : (i) the constant conditional correlation (CCC) model of Bollerslev (1990), (ii) the dynamic conditional correlation (DCC) model of Engle (2002), (iii) the asymmetric DCC (ASYDCC) of Cappiello *et al.* (2006), (iv) the dynamic equicorrelation (DECO) model of Engle and Kelly (2012). Engle and Colacito (2006) and Engle and Sheppard (2008) study the performance of alternative conditional correlation models in portfolio selection problems.

Multivariate GARCH models are typically estimated via quasi maximum likelihood (QML). However, this estimator is found to be severely biased in large dimensions; see, for instance, Engle *et al.* (2008) and Hafner and Reznikova (2012). In this paper, the parameters of the EWMA, ORE, and VECH specifications are estimated with the composite likelihood (CL) method proposed by Engle *et al.* (2008). As for the conditional correlation models, their estimation can be conveniently divided into volatility part and correlation part. The volatility part refers to estimating the univariate conditional variances which

is done by QML assuming Gaussian innovations. The parameters of the correlation matrix in the DCC and ASYDCC models are estimated using the CL method. As pointed out by Engle *et al.* (2008), the CL estimator provides more accurate parameter estimates in comparison to the two-step procedure proposed by Engle (2002), especially in large problems.

Finally, it is important to emphasize that other multivariate GARCH specifications, apart from those considered in this paper, have been proposed in the literature; see Bauwens *et al.* (2006) and Silvennoinen and Teräsvirta (2009) for reviews. In this sense, the model set considered in this paper can be increased in many alternative ways.³ Even though we consider these alternative specifications to be very interesting, we focus on a model set that includes 8 of the most widely used specifications. The computational effort to perform rolling window estimations of each of these 8 specifications for the high dimensional data sets considered in this paper is already immense, which limits to a large extent our capacity to increase the model set.

3 Empirical application

3.1 Data sets

We conduct an empirical investigation on the performance of both mean-variance and minimum variance portfolios obtained with the mean-var(δ, η) and min-var(δ, η) model combinations defined in (6) and (9), respectively. The investigation is based on daily observations of 4 alternative data sets from the global stock markets from 01/01/2004 until 31/12/2013. The first data set consists of closing prices of the 50 most traded stocks belonging to the US stock market index SP100 (50SP). The second data set consists of 100 US industry portfolios (100Ind) obtained from Ken French's web site. Both data sets have $T = 2517$ observations. The third data set consists of closing prices of 45 stocks belonging to the European stock market index Eurostoxx (45Euro). Finally, the fourth data set consists of closing prices of 17 stocks belonging to the Asian stock market index STI (17STI). These last two data sets have, respectively, $T = 2504$ and $T = 2520$ observations.

³Important contributions in the literature on multivariate volatility models include the regime switching DCC of Pelletier (2006), the Wishart stochastic volatility model of Philipov and Glickman (2006a,b), the semiparametric model for the correlation dynamics of Hafner *et al.* (2006), the generalized DCC model of Billio and Caporin (2009) and Hafner and Franses (2009), the factor-DCC model of Zhang and Chan (2009), the DCC-MIDAS of Colacito *et al.* (2011), the HEAVY models of Noureldin *et al.* (2012), the factor-spline-GARCH model of Rangel and Engle (2012), the heterogeneous ASYDCC model of Asai (2013), the multivariate rotated ARCH model of Noureldin *et al.* (2014), the realized beta GARCH model of Hansen *et al.* (2014), the dynamic factor multivariate GARCH model of Santos and Moura (2014), the smooth transition conditional correlation model of Silvennoinen and Teräsvirta (2015), among many others.

3.2 Methodology for evaluating portfolio performance and implementation details

In order to implement the individual models as well as the proposed mean-var(δ, η) and min-var(δ, η) model combinations, we employ an expanding window procedure that works as follows. Departing from the first $t = 1500$ observations, all models described in section 2.4 are estimated and their corresponding one-step-ahead forecasts of the conditional covariance matrix are obtained. We obtain mean-variance and minimum variance portfolio weights for each of the models along the t observations and compute the one-step-ahead forecast of the conditional combined conditional covariance estimator using the mean-var(δ, η) and min-var(δ, η) combination rule in (6) and (9), respectively, for $\delta = \{1, 0.95, 0.85, 0.80, 0.75\}$ and $\eta = \{0, 1, 2, 4, 10\}$. It is worth noting that when $\eta = 0$ the min-var(δ, η) combination rule corresponds to the equally weighted model combination irrespective of the value of δ . Finally, we add one observation to the estimation window and repeat the process until the end of the data set is reached. We end up with a sample of $T - t$ out-of-sample observations.

It is important to notice that the implementation procedure described above implies that the each candidate model is evaluated in terms of its in-sample portfolio performance. This choice is motivated by two reasons. First, it simplifies implementation as the same estimation window used to estimate the parameters of each individual model is also used to evaluate portfolio performance. Second, and most importantly, we performed extensive robustness checks to evaluate the gains (or lack of) from changing to past out-of-sample portfolio performance, despite higher computational cost. The results are very similar in comparison to those based on the past in-sample performance of each individual model, which makes us confident to employ this approach. The results based on an out-of-sample evaluation of individual models are available upon request.

We use the out-of-sample observations to evaluate the mean-variance and minimum variance portfolio performance in terms of average return ($\hat{\mu}$) and standard deviation (volatility) of returns ($\hat{\sigma}$). These statistics are calculated as follows:

$$\hat{\mu} = \frac{1}{T-1} \sum_{t=1}^{T-1} w'_t R_{t+1}$$

$$\hat{\sigma} = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T-1} (w'_t R_{t+1} - R_p)^2}.$$

An additional aspect that must be taken into account is the impact of transaction costs on the

performance of optimal portfolios (Han, 2006). For that purpose, we follow Della Corte *et al.* (2008) and Thornton and Valente (2012) and compute the portfolio return net of transaction costs ($R_{p,t}^{\text{net}}$). This calculation is performed as

$$R_{p,t}^{\text{net}} = (1 - c \cdot \text{turnover}_t) (1 + w_t' R_{t+1}) - 1, \quad (10)$$

where c is the fee that must be paid for each transaction and turnover_t is the portfolio turnover at time t , defined as the fraction of wealth traded between periods t and $t + 1$, i.e

$$\text{turnover}_t = \sum_{j=1}^N (|w_{j,t+1} - w_{j,t}|).$$

We follow Kirby and Ostdiek (2012) and evaluate the portfolio performance considering alternative levels for the transaction cost fee c . More specifically, we consider the instances in which $c = 0$ basis points (bp), i.e. no transaction costs involved, $c = 10$ bp and $c = 20$ bp. Upon computing the average and the standard deviation of the portfolio return net of transaction costs for each level of c , we report the average portfolio turnover over all out-of-sample observations as well as the risk-adjusted net return measured by the Sharpe ratio, which is defined as

$$SR = \frac{\bar{R}_p^{\text{net}}}{\sigma^{\text{net}}}, \quad (11)$$

where \bar{R}_p^{net} and σ^{net} are, respectively, the average and the standard deviation of portfolio returns net of transaction costs.

In order to assess the relative performance of the model combination approach proposed in the paper, we consider as the main benchmark specification the min-var($\eta = 0$) combination, which corresponds to the equally weighted model combination. The stationary bootstrap of Politis and Romano (1994) with $B=1000$ resamples and block size $b = 5$ was used to test the statistical significance of differences between the standard deviation of mean-variance and minimum variance portfolio returns obtained with each individual model and with each model combination with respect to those obtained with the benchmark.⁴ The methodology suggested in Ledoit and Wolf (2008) was used to obtain p -values.

⁴We performed extensive robustness checks regarding the choice of the block size. More specifically, we performed the test with the block size varying from 1 to 100. The results are very similar to those reported here. Moreover, we also performed the test of the statistical significance of differences in the variance of minimum variance portfolio returns. The results are also similar to those obtained with the standard deviation and are available upon request.

3.3 Results

Table 1 to 4 report for each of the data sets considered in the paper performance statistics for the mean-variance and minimum variance portfolios obtained with each of the individual models as well as with their combinations according to the mean-var(δ, η) and min-var(δ, η) criteria, respectively. In each Table, the left-hand side reports the performance statistics for the mean-variance portfolios, whereas the right-hand side reports the figures for the minimum variance portfolios.

Let us initially consider the performance of the minimum variance portfolios obtained with individual multivariate GARCH models as well as with their min-var(δ, η) combinations. The first striking result in Tables 1 to 4 is that all individual models delivered minimum variance portfolios with higher risk with respect to those obtained with the model combinations. This result is true for all data sets. First, when comparing the individual models with respect to benchmark, we observe that a simple model averaging is able to deliver less risky minimum variance portfolio with respect to the best single model in all cases, and that these differences are statistically significant. Second, when comparing among alternative model combinations, we find that varying the δ and η parameters bring improvements in many instances. We observe that many alternative model combinations outperform the simple model averaging. In general, lowering the value of δ (i.e. putting more weight on the recent performance of individual models) and increasing the value of η (i.e. putting more weight on the best single models) lead to improvement with respect to the min-var($\eta = 0$) specification in terms of portfolio risk. The best overall result in all data sets in terms of standard deviation of portfolio returns is achieved by the min-var($\delta=0.75, \eta=10$) combination. On the other hand, we observe that putting more weight on the recent performance of individual models and putting more weight on the best single models tend to increase substantially the portfolio turnover. The comparison of other performance statistics such as Sharpe ratios also suggest that min-var(δ, η) model combinations perform at least equivalently with respect to individual models. These numbers suggest that the proposed model combinations is able to not deliver less risky portfolio but also attractive risk-adjusted returns and portfolio turnover that are at least equivalent to those obtained by single models.

We now turn our attention to the analysis of the mean-variance portfolios obtained with individual multivariate GARCH models as well as with their mean-var(δ, η) combinations. The main concern in this case is on the risk-adjusted performance of the resulting mean-variance portfolios, specially when transaction costs are taken into account. We observe in Tables 1 to 4 that, in the absence of transaction costs, the vast majority of mean-var(δ, η) combinations yield statistically higher SR in comparison to not

only the equally weighted combination but also higher than those obtained with individual models. Even more importantly, this result also holds when transaction costs are taken into account. We observe that this result is more evident when the level of transaction costs is below 20 b.p. For instance, in the case of the 50SP data set, the SR of mean-variance portfolios based on portfolio returns net of transaction costs of 10 b.p. is 0.205 for the mean-var($\delta = 0.80, \eta = 10$) combination, whereas the best result obtained with an individual model is 0.121. Similar results are obtained with the remaining data sets.

Finally, as an illustration, we report in Table 5 the average and standard deviation of combination weights for each min-var(δ, η) specification. As expected, we observe that the best performing models tend to receive more weights in the model combination. This result is very much evident specially for the EWMA, ORE, and CCC specifications as they have weights higher than 20% in many instances.

4 Concluding remarks

Modeling and forecasting the covariance matrix of portfolio returns is of paramount importance in many economic and financial problems such as asset pricing, portfolio optimization and market risk management. In practice, however, the investor faces uncertainty on which is the most appropriate specification to model and to perform these tasks. To alleviate this problem, we put forward a novel approach to combine multivariate volatility predictions from alternative conditional covariance models. The proposed combination rule is explicitly built to exploit model complementarities and is motivated by the fact this class of models is often applied in portfolio selection problems. Four major advantages of this approach are that i) does not require a proxy for the latent conditional covariance matrix, ii) does not require optimization of the combination weights, iii) holds the equally-weighted model combination as a particular case, and iv) accommodates alternative portfolio policies as well as alternative portfolio characteristics for defining the mixing. Our empirical evidence based on 4 data sets with cross section dimensions ranging from 17 to 100 assets over a 10-year time span confirms that mean-variance and minimum variance portfolios obtained with the combined conditional covariance estimators have improved performance in terms of risk-adjusted returns and are substantially and statistically less risky with respect to all individual models considered. The results are robust to the presence of transaction costs.

Table 1: Mean-variance and minimum variance portfolio performance: individual models vs. model combinations for the SP50 data set

The Table reports performance statistics for mean-variance and minimum variance portfolios obtained with a set of individual models as well as with model combinations based on the mean-var(δ, η) and min-var(δ, η) combination rules in (6) and (9), respectively, for $\delta = \{1, 0.95, 0.85, 0.80, 0.75\}$ and $\eta = \{0, 1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VECM, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations from 01/01/2004 until 31/12/2013. SR denotes the Sharpe ratio, which is computed using alternative levels of the transaction cost fee c : $c=0$ basis points (b.p.), $c=10$ b.p., and $c=20$ b.p. The data set consists of closing prices of the 50 mostly traded stocks belonging to the US stock market index SP100 (50SP). All figures are based on out-of-sample daily observations. Optimal portfolios are re-balanced on a daily basis.

	Mean return (%)	Std. dev. (%)	Turnover	SR (c=0 b.p.)	SR (c=10 b.p.)	SR (c=20 b.p.)	Mean return (%)	Std. dev. (%)	Turnover	SR (c=0 b.p.)	SR (c=10 b.p.)	SR (c=20 b.p.)
	Mean-variance portfolios						Minimum variance portfolios					
<i>Individual models</i>												
EWMA	0.048	0.468	0.111	0.103	0.079	0.055	0.049	0.467	0.111	0.105	0.081	0.057
ORE	0.051	0.455	0.063	0.111	0.097	0.083	0.051	0.453	0.062	0.113	0.099	0.086
VECH	0.045	0.524	0.299	0.085	0.027	-0.030	0.044	0.524	0.297	0.084	0.027	-0.029
O-GARCH	0.058	0.451	0.036	0.129	0.121	0.113	0.056	0.457	0.033	0.123	0.116	0.109
CCC	0.043	0.483	0.291	0.090	0.030	-0.031	0.042	0.481	0.284	0.087	0.028	-0.031
DCC	0.048	0.854	1.096	0.057	-0.072	-0.198	0.047	0.876	1.128	0.054	-0.075	-0.202
DECO	0.039	0.501	0.181	0.078	0.042	0.006	0.037	0.501	0.181	0.074	0.038	0.002
ASY-DCC	0.046	0.479	0.223	0.095	0.049	0.002	0.046	0.476	0.221	0.097	0.050	0.004
<i>Model combinations</i>												
$\delta=1, \eta=0$ (benchmark)	0.047	0.441	0.152	0.106	0.071	0.037	0.046	0.441	0.151	0.105	0.070	0.036
$\delta=1, \eta=1$	0.048	0.441	0.153	0.110	0.075	0.040	0.047	0.437	0.132	0.108	0.077	0.047
$\delta=1, \eta=2$	0.049	0.442	0.154	0.111	0.076	0.041	0.047	0.436	0.127	0.109	0.079	0.050
$\delta=1, \eta=4$	0.049	0.445	0.158	0.111	0.075	0.039	0.047	0.437	0.132	0.108	0.078	0.047
$\delta=1, \eta=10$	0.047	0.457	0.166	0.103	0.067	0.031	0.045	0.447	0.180	0.102	0.061	0.021
$\delta=0.95, \eta=1$	0.063	0.443	0.188	0.141	0.099	0.056	0.047	0.432	0.125	0.109	0.080	0.051
$\delta=0.95, \eta=2$	0.074	0.449	0.236	0.166	0.113	0.060	0.047	0.426	0.116	0.111	0.084	0.056
$\delta=0.95, \eta=4$	0.090	0.461	0.311	0.194	0.127	0.059	0.047	0.418	0.114	0.113	0.085	0.058
$\delta=0.95, \eta=10$	0.109	0.484	0.406	0.225	0.141	0.056	0.045	0.407	0.136	0.111	0.077	0.044
$\delta=0.85, \eta=1$	0.076	0.443	0.235	0.171	0.117	0.064	0.046	0.424	0.135	0.109	0.077	0.045
$\delta=0.85, \eta=2$	0.098	0.446	0.339	0.220	0.144	0.067	0.046	0.413	0.144	0.112	0.076	0.041
$\delta=0.85, \eta=4$	0.125	0.457	0.468	0.273	0.171	0.067	0.045	0.396	0.178	0.114	0.069	0.024
$\delta=0.85, \eta=10$	0.148	0.476	0.578	0.311	0.191	0.067	0.044	0.376	0.255	0.117	0.049	-0.018
$\delta=0.80, \eta=1$	0.080	0.443	0.256	0.181	0.123	0.065	0.046	0.421	0.143	0.109	0.075	0.041
$\delta=0.80, \eta=2$	0.106	0.446	0.381	0.238	0.152	0.066	0.046	0.406	0.162	0.112	0.072	0.032
$\delta=0.80, \eta=4$	0.138	0.459	0.532	0.300	0.185	0.067	0.045	0.387	0.212	0.117	0.061	0.007
$\delta=0.80, \eta=10$	0.164	0.484	0.659	0.339	0.205	0.065	0.044	0.366	0.308	0.119	0.035	-0.048
$\delta=0.75, \eta=1$	0.083	0.444	0.274	0.187	0.125	0.063	0.046	0.418	0.152	0.110	0.073	0.037
$\delta=0.75, \eta=2$	0.111	0.448	0.416	0.249	0.155	0.062	0.046	0.401	0.183	0.114	0.068	0.022
$\delta=0.75, \eta=4$	0.145	0.463	0.588	0.313	0.186	0.057	0.045	0.379	0.248	0.119	0.053	-0.013
$\delta=0.75, \eta=10$	0.170	0.483	0.726	0.353	0.204	0.050	0.042	0.357	0.364	0.119	0.016	-0.084

Table 2: Mean-variance and minimum variance portfolio performance: individual models vs. model combinations for the 100Ind data set

The Table reports performance statistics for mean-variance and minimum variance portfolios obtained with a set of individual models as well as with model combinations based on the mean-var(δ, η) and min-var(δ, η) combination rules in (6) and (9), respectively, for $\delta = \{1, 0.95, 0.85, 0.80, 0.75\}$ and $\eta = \{0, 1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VECM, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations from 01/01/2004 until 31/12/2013. SR denotes the Sharpe ratio, which is computed using alternative levels of the transaction cost fee c : $c = 0$ basis points (b.p.), $c = 10$ b.p., and $c = 20$ b.p. The data set consists of 100 US industry portfolios (100Ind) obtained from Ken French's web site. All figures are based on out-of-sample daily observations. Optimal portfolios are re-balanced on a daily basis.

	Mean return (%)	Std. dev. (%)	Turnover	SR (c=0 b.p.)	SR (c=10 b.p.)	SR (c=20 b.p.)	Mean return (%)	Std. dev. (%)	Turnover	SR (c=0 b.p.)	SR (c=10 b.p.)	SR (c=20 b.p.)
	Mean-variance portfolios						Minimum variance portfolios					
<i>Individual models</i>												
EWMA	0.080	0.563	0.684	0.141	0.020	-0.099	0.080	0.562	0.680	0.143	0.022	-0.096
ORE	0.069	0.633	0.085	0.110	0.097	0.083	0.069	0.631	0.075	0.110	0.098	0.086
VECH	0.077	0.612	1.288	0.125	-0.084	-0.281	0.077	0.606	1.265	0.127	-0.080	-0.275
O-GARCH	0.056	0.724	0.098	0.077	0.064	0.051	0.055	0.728	0.090	0.076	0.064	0.052
CCC	0.065	0.583	1.146	0.112	-0.083	-0.272	0.065	0.581	1.139	0.112	-0.082	-0.270
DCC	0.042	1.010	4.650	0.042	-0.392	-0.716	0.044	1.011	4.599	0.044	-0.384	-0.703
DECO	0.056	0.688	0.645	0.081	-0.011	-0.104	0.057	0.674	0.629	0.085	-0.007	-0.100
ASY-DCC	0.059	0.833	1.722	0.071	-0.134	-0.329	0.058	0.829	1.704	0.070	-0.134	-0.327
<i>Model combinations</i>												
$\delta=1, \eta=0$ (benchmark)	0.069	0.509	0.683	0.137	0.003	-0.128	0.069	0.506	0.675	0.136	0.003	-0.127
$\delta=1, \eta=1$	0.070	0.513	0.520	0.137	0.036	-0.064	0.070	0.510	0.586	0.137	0.022	-0.091
$\delta=1, \eta=2$	0.071	0.520	0.455	0.136	0.049	-0.038	0.070	0.515	0.540	0.137	0.032	-0.072
$\delta=1, \eta=4$	0.072	0.533	0.378	0.134	0.064	-0.007	0.071	0.523	0.514	0.136	0.038	-0.059
$\delta=1, \eta=10$	0.072	0.577	0.213	0.126	0.089	0.052	0.073	0.541	0.526	0.135	0.038	-0.059
$\delta=0.95, \eta=1$	0.110	0.499	1.019	0.221	0.017	-0.176	0.070	0.493	0.633	0.142	0.014	-0.111
$\delta=0.95, \eta=2$	0.145	0.513	1.427	0.283	0.005	-0.244	0.071	0.483	0.645	0.147	0.014	-0.117
$\delta=0.95, \eta=4$	0.177	0.567	1.907	0.312	-0.023	-0.302	0.070	0.467	0.752	0.150	-0.010	-0.166
$\delta=0.95, \eta=10$	0.214	0.628	2.638	0.341	-0.072	-0.369	0.063	0.457	1.071	0.139	-0.093	-0.310
$\delta=0.85, \eta=1$	0.137	0.494	1.335	0.277	0.007	-0.243	0.070	0.479	0.707	0.146	-0.001	-0.146
$\delta=0.85, \eta=2$	0.187	0.501	1.976	0.374	-0.019	-0.350	0.071	0.456	0.859	0.155	-0.033	-0.215
$\delta=0.85, \eta=4$	0.236	0.527	2.730	0.447	-0.064	-0.419	0.069	0.423	1.231	0.164	-0.125	-0.388
$\delta=0.85, \eta=10$	0.296	0.560	3.725	0.528	-0.115	-0.479	0.064	0.403	1.853	0.158	-0.282	-0.601
$\delta=0.80, \eta=1$	0.144	0.493	1.488	0.293	-0.008	-0.283	0.069	0.472	0.760	0.147	-0.014	-0.171
$\delta=0.80, \eta=2$	0.200	0.494	2.245	0.404	-0.047	-0.415	0.070	0.445	0.995	0.157	-0.066	-0.280
$\delta=0.80, \eta=4$	0.254	0.517	3.084	0.491	-0.094	-0.482	0.068	0.406	1.494	0.167	-0.195	-0.509
$\delta=0.80, \eta=10$	0.318	0.554	4.121	0.575	-0.145	-0.547	0.062	0.383	2.222	0.162	-0.379	-0.717
$\delta=0.75, \eta=1$	0.152	0.492	1.611	0.308	-0.018	-0.313	0.069	0.467	0.822	0.147	-0.028	-0.200
$\delta=0.75, \eta=2$	0.212	0.491	2.454	0.432	-0.064	-0.459	0.069	0.435	1.144	0.158	-0.104	-0.352
$\delta=0.75, \eta=4$	0.270	0.503	3.370	0.538	-0.116	-0.524	0.066	0.392	1.757	0.167	-0.270	-0.626
$\delta=0.75, \eta=10$	0.332	0.540	4.376	0.615	-0.163	-0.573	0.059	0.367	2.561	0.161	-0.471	-0.813

Table 3: Mean-variance and minimum variance portfolio performance: individual models vs. model combinations for the 45Euro data set

The Table reports performance statistics for mean-variance and minimum variance portfolios obtained with a set of individual models as well as with model combinations based on the mean-var(δ, η) and min-var(δ, η) combination rules in (6) and (9), respectively, for $\delta = \{1, 0.95, 0.85, 0.80, 0.75\}$ and $\eta = \{0, 1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VECH, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations from 01/01/2004 until 31/12/2013. SR denotes the Sharpe ratio, which is computed using alternative levels of the transaction cost fee c : $c=0$ basis points (b.p.), $c=10$ b.p., and $c=20$ b.p. The data set consists of closing prices of 45 stocks belonging to the European stock market index Eurostoxx (45Euro). All figures are based on out-of-sample daily observations. Optimal portfolios are re-balanced on a daily basis.

	Mean return (%)	Std. dev. (%)	Turnover	SR (c=0 b.p.)	SR (c=10 b.p.)	SR (c=20 b.p.)	Mean return (%)	Std. dev. (%)	Turnover	SR (c=0 b.p.)	SR (c=10 b.p.)	SR (c=20 b.p.)
	Mean-variance portfolios						Minimum variance portfolios					
<i>Individual models</i>												
EWMA	0.058	0.816	0.214	0.071	0.046	0.020	0.060	0.814	0.212	0.074	0.049	0.023
ORE	0.057	0.812	0.104	0.070	0.059	0.046	0.057	0.810	0.100	0.070	0.060	0.047
VECH	0.057	0.844	0.557	0.067	0.003	-0.063	0.054	0.843	0.549	0.064	0.000	-0.065
O-GARCH	0.034	0.954	0.105	0.035	0.026	0.015	0.032	0.952	0.101	0.033	0.024	0.013
CCC	0.060	0.814	0.496	0.074	0.014	-0.047	0.060	0.815	0.478	0.074	0.017	-0.042
DCC	0.023	1.452	2.748	0.016	-0.171	-0.349	0.004	1.460	2.734	0.002	-0.182	-0.356
DECO	0.038	0.865	0.416	0.044	-0.002	-0.050	0.044	0.909	0.399	0.048	0.005	-0.038
ASY-DCC	0.059	0.841	0.560	0.071	0.006	-0.061	0.056	0.839	0.556	0.067	0.002	-0.064
<i>Model combinations</i>												
$\delta=1, \eta=0$ (benchmark)	0.055	0.754	0.346	0.073	0.029	-0.017	0.054	0.759	0.346	0.072	0.028	-0.018
$\delta=1, \eta=1$	0.057	0.752	0.309	0.076	0.037	-0.004	0.056	0.757	0.325	0.074	0.033	-0.010
$\delta=1, \eta=2$	0.059	0.755	0.326	0.077	0.036	-0.007	0.057	0.756	0.317	0.076	0.036	-0.006
$\delta=1, \eta=4$	0.059	0.771	0.380	0.077	0.029	-0.020	0.059	0.757	0.321	0.079	0.038	-0.005
$\delta=1, \eta=10$	0.061	0.803	0.472	0.076	0.018	-0.040	0.062	0.769	0.371	0.081	0.034	-0.014
$\delta=0.95, \eta=1$	0.101	0.763	0.495	0.132	0.069	0.004	0.056	0.748	0.312	0.075	0.034	-0.007
$\delta=0.95, \eta=2$	0.135	0.769	0.676	0.176	0.090	0.002	0.056	0.739	0.306	0.076	0.037	-0.005
$\delta=0.95, \eta=4$	0.169	0.794	0.948	0.213	0.095	-0.024	0.057	0.726	0.318	0.078	0.036	-0.008
$\delta=0.95, \eta=10$	0.204	0.845	1.258	0.242	0.094	-0.053	0.057	0.706	0.376	0.081	0.029	-0.024
$\delta=0.85, \eta=1$	0.128	0.769	0.594	0.166	0.090	0.013	0.057	0.731	0.332	0.079	0.035	-0.010
$\delta=0.85, \eta=2$	0.179	0.785	0.871	0.228	0.118	0.007	0.059	0.710	0.361	0.083	0.034	-0.017
$\delta=0.85, \eta=4$	0.242	0.817	1.316	0.296	0.136	-0.024	0.060	0.680	0.442	0.088	0.025	-0.040
$\delta=0.85, \eta=10$	0.310	0.885	1.771	0.350	0.152	-0.048	0.056	0.643	0.620	0.088	-0.007	-0.102
$\delta=0.80, \eta=1$	0.138	0.770	0.631	0.179	0.098	0.016	0.058	0.724	0.347	0.080	0.034	-0.014
$\delta=0.80, \eta=2$	0.194	0.787	0.937	0.246	0.128	0.010	0.059	0.697	0.399	0.085	0.029	-0.028
$\delta=0.80, \eta=4$	0.262	0.816	1.399	0.320	0.150	-0.021	0.059	0.662	0.517	0.089	0.013	-0.064
$\delta=0.80, \eta=10$	0.328	0.871	1.842	0.377	0.166	-0.044	0.055	0.622	0.738	0.089	-0.028	-0.144
$\delta=0.75, \eta=1$	0.148	0.769	0.667	0.192	0.107	0.020	0.058	0.717	0.365	0.081	0.031	-0.020
$\delta=0.75, \eta=2$	0.209	0.784	0.998	0.266	0.140	0.013	0.059	0.686	0.442	0.086	0.023	-0.041
$\delta=0.75, \eta=4$	0.277	0.814	1.483	0.340	0.159	-0.023	0.058	0.647	0.595	0.089	-0.001	-0.092
$\delta=0.75, \eta=10$	0.341	0.866	1.883	0.394	0.178	-0.039	0.053	0.603	0.847	0.087	-0.051	-0.187

Table 4: Mean-variance and minimum variance portfolio performance: individual models vs. model combinations for the 17STI data set

The Table reports performance statistics for mean-variance and minimum variance portfolios obtained with a set of individual models as well as with model combinations based on the mean-var(δ, η) and min-var(δ, η) combination rules in (6) and (9), respectively, for $\delta = \{1, 0.95, 0.85, 0.80, 0.75\}$ and $\eta = \{0, 1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VEC, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations from 01/01/2004 until 31/12/2013. SR denotes the Sharpe ratio, which is computed using alternative levels of the transaction cost fee c : $c=0$ basis points (b.p.), $c=10$ b.p., and $c=20$ b.p. The data set consists of closing prices of 17 stocks belonging to the Asian stock market index STI (17STI). All figures are based on out-of-sample daily observations. Optimal portfolios are re-balanced on a daily basis.

	Mean return (%)	Std. dev. (%)	Turnover	SR (c=0 b.p.)	SR (c=10 b.p.)	SR (c=20 b.p.)	Mean return (%)	Std. dev. (%)	Turnover	SR (c=0 b.p.)	SR (c=10 b.p.)	SR (c=20 b.p.)
	Mean-variance portfolios						Minimum variance portfolios					
<i>Individual models</i>												
EWMA	0.033	0.727	0.061	0.045	0.036	0.028	0.033	0.705	0.060	0.046	0.037	0.029
ORE	0.033	0.729	0.057	0.045	0.037	0.029	0.033	0.707	0.056	0.047	0.038	0.030
VECH	0.035	0.734	0.213	0.048	0.018	-0.011	0.033	0.703	0.207	0.047	0.017	-0.013
O-GARCH	0.031	0.756	0.053	0.042	0.034	0.027	0.031	0.727	0.050	0.042	0.035	0.028
CCC	0.045	0.710	0.197	0.063	0.035	0.007	0.042	0.689	0.205	0.061	0.031	0.001
DCC	0.035	1.154	1.246	0.030	-0.078	-0.184	0.042	1.154	1.214	0.037	-0.068	-0.170
DECO	0.031	0.724	0.176	0.043	0.019	-0.006	0.023	0.732	0.171	0.031	0.007	-0.016
ASY-DCC	0.041	0.715	0.145	0.057	0.035	0.015	0.039	0.699	0.149	0.056	0.034	0.013
<i>Model combinations</i>												
$\delta=1, \eta=0$ (benchmark)	0.038	0.698	0.147	0.054	0.032	0.011	0.035	0.671	0.146	0.052	0.030	0.008
$\delta=1, \eta=1$	0.038	0.696	0.124	0.055	0.037	0.019	0.036	0.670	0.122	0.053	0.034	0.016
$\delta=1, \eta=2$	0.039	0.696	0.115	0.055	0.038	0.022	0.036	0.670	0.114	0.054	0.036	0.019
$\delta=1, \eta=4$	0.039	0.696	0.112	0.056	0.039	0.023	0.037	0.669	0.113	0.055	0.037	0.020
$\delta=1, \eta=10$	0.040	0.699	0.119	0.057	0.039	0.022	0.038	0.670	0.126	0.057	0.037	0.019
$\delta=0.95, \eta=1$	0.054	0.700	0.197	0.077	0.048	0.020	0.034	0.662	0.124	0.052	0.033	0.014
$\delta=0.95, \eta=2$	0.069	0.710	0.287	0.097	0.055	0.015	0.034	0.656	0.117	0.051	0.033	0.015
$\delta=0.95, \eta=4$	0.084	0.734	0.393	0.114	0.060	0.007	0.032	0.647	0.117	0.050	0.031	0.013
$\delta=0.95, \eta=10$	0.101	0.751	0.483	0.134	0.069	0.005	0.029	0.632	0.137	0.046	0.024	0.002
$\delta=0.85, \eta=1$	0.072	0.702	0.234	0.102	0.069	0.035	0.033	0.651	0.131	0.051	0.030	0.010
$\delta=0.85, \eta=2$	0.101	0.716	0.357	0.141	0.091	0.041	0.031	0.636	0.136	0.048	0.026	0.005
$\delta=0.85, \eta=4$	0.133	0.736	0.503	0.181	0.112	0.043	0.026	0.614	0.161	0.043	0.016	-0.010
$\delta=0.85, \eta=10$	0.159	0.760	0.626	0.210	0.127	0.044	0.019	0.584	0.227	0.033	-0.007	-0.046
$\delta=0.80, \eta=1$	0.077	0.702	0.249	0.110	0.074	0.038	0.032	0.645	0.136	0.050	0.028	0.007
$\delta=0.80, \eta=2$	0.110	0.718	0.389	0.153	0.098	0.044	0.029	0.626	0.149	0.047	0.022	-0.002
$\delta=0.80, \eta=4$	0.142	0.739	0.546	0.192	0.117	0.043	0.024	0.600	0.189	0.040	0.007	-0.024
$\delta=0.80, \eta=10$	0.173	0.758	0.686	0.229	0.137	0.046	0.017	0.570	0.275	0.030	-0.019	-0.067
$\delta=0.75, \eta=1$	0.081	0.704	0.266	0.115	0.077	0.039	0.032	0.640	0.142	0.050	0.027	0.005
$\delta=0.75, \eta=2$	0.116	0.717	0.417	0.161	0.102	0.044	0.028	0.617	0.165	0.046	0.018	-0.008
$\delta=0.75, \eta=4$	0.148	0.737	0.584	0.201	0.122	0.042	0.022	0.587	0.220	0.038	0.000	-0.037
$\delta=0.75, \eta=10$	0.180	0.764	0.729	0.236	0.140	0.044	0.019	0.554	0.321	0.033	-0.025	-0.082

Table 5: **Combination weights: min-var(δ, η) combination rule**

The Table reports the average and the standard deviation of the min-var(δ, η) combination weights for $\delta = \{1, 0.95, 0.85\}$ and $\eta = \{1, 2, 4, 10\}$. The individual models are the following multivariate GARCH specifications: exponentially weighted moving average (EWMA), optimal rolling estimator (ORE), VECH, orthogonal GARCH (O-GARCH), constant conditional correlation (CCC), dynamic conditional correlation (DCC), dynamic equicorrelation (DECO), and asymmetric DCC (ASYDCC). The figures are based on daily observations of 4 data sets from the global stock markets from 01/01/2004 until 31/12/2013. The first data set consists of closing prices of the 50 mostly traded stocks belonging to the US stock market index SandP100 (50SandP). The second data set consists of 100 US industry portfolios (100Ind) obtained from Ken French's web site. The third data set consists of closing prices of 45 stocks belonging to the European stock market index Eurostoxx (45Euro). Finally, the fourth data sets consists of closing prices of 17 stocks belonging to the Asian stock market index STI (17STI). All figures are based on out-of-sample daily observations.

	EWMA		ORE		VECH		OGARCH		CCC		DCC		DECO		ASYDCC	
	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.	Avg. weight	Std. dev.
Data set: 50SandP																
min-var($\delta=1, \eta=1$)	0.135	0.000	0.148	0.001	0.099	0.002	0.124	0.002	0.154	0.001	0.053	0.002	0.139	0.001	0.149	0.001
min-var($\delta=1, \eta=2$)	0.136	0.001	0.165	0.002	0.074	0.003	0.115	0.003	0.178	0.002	0.021	0.001	0.144	0.001	0.167	0.003
min-var($\delta=1, \eta=4$)	0.128	0.002	0.187	0.005	0.037	0.003	0.092	0.005	0.219	0.005	0.003	0.000	0.143	0.002	0.192	0.007
min-var($\delta=1, \eta=10$)	0.083	0.004	0.216	0.020	0.004	0.001	0.037	0.006	0.319	0.014	0.000	0.000	0.111	0.004	0.230	0.016
min-var($\delta=0.95, \eta=1$)	0.135	0.018	0.149	0.020	0.123	0.021	0.140	0.028	0.145	0.018	0.045	0.013	0.127	0.020	0.136	0.017
min-var($\delta=0.95, \eta=2$)	0.137	0.036	0.166	0.044	0.114	0.039	0.149	0.059	0.157	0.038	0.016	0.010	0.122	0.037	0.139	0.036
min-var($\delta=0.95, \eta=4$)	0.133	0.065	0.192	0.099	0.066	0.065	0.163	0.119	0.170	0.079	0.002	0.004	0.107	0.066	0.136	0.073
min-var($\delta=0.95, \eta=10$)	0.115	0.120	0.244	0.240	0.066	0.115	0.187	0.221	0.184	0.180	0.000	0.000	0.079	0.113	0.125	0.155
min-var($\delta=0.85, \eta=1$)	0.134	0.031	0.147	0.033	0.124	0.034	0.141	0.044	0.145	0.031	0.048	0.024	0.126	0.033	0.136	0.030
min-var($\delta=0.85, \eta=2$)	0.134	0.059	0.161	0.068	0.118	0.065	0.152	0.091	0.155	0.065	0.021	0.025	0.120	0.063	0.139	0.063
min-var($\delta=0.85, \eta=4$)	0.128	0.104	0.177	0.135	0.107	0.114	0.169	0.169	0.165	0.128	0.007	0.026	0.109	0.110	0.138	0.123
min-var($\delta=0.85, \eta=10$)	0.113	0.180	0.194	0.254	0.092	0.189	0.189	0.278	0.176	0.247	0.004	0.040	0.092	0.179	0.139	0.230
Data set: 100Ind																
min-var($\delta=1, \eta=1$)	0.149	0.006	0.187	0.015	0.135	0.005	0.126	0.004	0.183	0.002	0.060	0.001	0.112	0.003	0.047	0.004
min-var($\delta=1, \eta=2$)	0.155	0.015	0.244	0.035	0.128	0.011	0.110	0.005	0.234	0.004	0.025	0.001	0.088	0.007	0.016	0.003
min-var($\delta=1, \eta=4$)	0.139	0.032	0.340	0.080	0.094	0.020	0.068	0.003	0.310	0.022	0.004	0.001	0.044	0.008	0.001	0.001
min-var($\delta=1, \eta=10$)	0.064	0.037	0.490	0.185	0.023	0.013	0.009	0.002	0.410	0.133	0.000	0.000	0.004	0.002	0.000	0.000
min-var($\delta=0.95, \eta=1$)	0.170	0.030	0.131	0.023	0.143	0.025	0.114	0.025	0.173	0.041	0.055	0.019	0.124	0.023	0.089	0.035
min-var($\delta=0.95, \eta=2$)	0.209	0.071	0.125	0.044	0.149	0.052	0.096	0.042	0.219	0.095	0.024	0.017	0.112	0.042	0.065	0.059
min-var($\delta=0.95, \eta=4$)	0.271	0.159	0.099	0.076	0.139	0.093	0.065	0.061	0.297	0.205	0.005	0.008	0.082	0.067	0.041	0.087
min-var($\delta=0.95, \eta=10$)	0.369	0.325	0.056	0.131	0.090	0.128	0.030	0.076	0.394	0.367	0.000	0.000	0.038	0.090	0.023	0.101
min-var($\delta=0.85, \eta=1$)	0.165	0.047	0.129	0.035	0.143	0.044	0.115	0.040	0.175	0.062	0.058	0.029	0.124	0.043	0.092	0.049
min-var($\delta=0.85, \eta=2$)	0.196	0.104	0.119	0.066	0.148	0.089	0.100	0.073	0.221	0.137	0.029	0.031	0.114	0.082	0.072	0.084
min-var($\delta=0.85, \eta=4$)	0.233	0.207	0.095	0.110	0.147	0.156	0.079	0.123	0.284	0.258	0.010	0.028	0.095	0.134	0.056	0.126
min-var($\delta=0.85, \eta=10$)	0.262	0.340	0.070	0.175	0.136	0.239	0.066	0.187	0.342	0.391	0.003	0.031	0.072	0.190	0.049	0.172
Data set: 45Euro																
min-var($\delta=1, \eta=1$)	0.132	0.002	0.139	0.002	0.125	0.002	0.108	0.002	0.163	0.002	0.067	0.004	0.125	0.001	0.142	0.001
min-var($\delta=1, \eta=2$)	0.133	0.004	0.147	0.004	0.119	0.003	0.089	0.002	0.204	0.004	0.034	0.005	0.120	0.002	0.154	0.001
min-var($\delta=1, \eta=4$)	0.125	0.007	0.153	0.007	0.099	0.005	0.056	0.003	0.292	0.013	0.008	0.002	0.101	0.003	0.166	0.003
min-var($\delta=1, \eta=10$)	0.071	0.012	0.116	0.018	0.040	0.007	0.009	0.001	0.582	0.038	0.000	0.000	0.041	0.002	0.141	0.007
min-var($\delta=0.95, \eta=1$)	0.148	0.021	0.149	0.021	0.139	0.019	0.111	0.029	0.149	0.025	0.047	0.019	0.118	0.022	0.139	0.015
min-var($\delta=0.95, \eta=2$)	0.162	0.043	0.164	0.042	0.143	0.039	0.096	0.048	0.167	0.054	0.019	0.017	0.106	0.039	0.143	0.030
min-var($\delta=0.95, \eta=4$)	0.179	0.091	0.182	0.084	0.143	0.076	0.076	0.069	0.194	0.120	0.005	0.010	0.082	0.063	0.139	0.058
min-var($\delta=0.95, \eta=10$)	0.202	0.205	0.212	0.194	0.134	0.163	0.053	0.102	0.244	0.263	0.000	0.001	0.044	0.091	0.111	0.118
min-var($\delta=0.85, \eta=1$)	0.145	0.032	0.147	0.032	0.137	0.030	0.113	0.040	0.151	0.041	0.050	0.029	0.120	0.038	0.138	0.029
min-var($\delta=0.85, \eta=2$)	0.155	0.066	0.159	0.064	0.139	0.061	0.101	0.068	0.170	0.087	0.024	0.030	0.112	0.072	0.140	0.058
min-var($\delta=0.85, \eta=4$)	0.163	0.127	0.170	0.121	0.136	0.115	0.090	0.111	0.197	0.169	0.010	0.028	0.100	0.122	0.134	0.109
min-var($\delta=0.85, \eta=10$)	0.166	0.232	0.176	0.228	0.131	0.213	0.083	0.188	0.233	0.300	0.004	0.023	0.085	0.189	0.122	0.201
Data set: 17STI																
min-var($\delta=1, \eta=1$)	0.134	0.000	0.135	0.000	0.128	0.001	0.132	0.000	0.150	0.001	0.048	0.000	0.134	0.001	0.140	0.000
min-var($\delta=1, \eta=2$)	0.136	0.001	0.138	0.001	0.124	0.002	0.131	0.001	0.170	0.001	0.017	0.000	0.136	0.002	0.148	0.000
min-var($\delta=1, \eta=4$)	0.132	0.002	0.137	0.002	0.109	0.003	0.123	0.001	0.206	0.003	0.002	0.000	0.133	0.003	0.156	0.001
min-var($\delta=1, \eta=10$)	0.110	0.004	0.119	0.004	0.068	0.005	0.092	0.003	0.332	0.009	0.000	0.000	0.112	0.005	0.167	0.002
min-var($\delta=0.95, \eta=1$)	0.135	0.015	0.134	0.016	0.138	0.013	0.132	0.022	0.141	0.015	0.057	0.021	0.125	0.020	0.138	0.012
min-var($\delta=0.95, \eta=2$)	0.138	0.029	0.137	0.030	0.144	0.027	0.135	0.043	0.151	0.032	0.028	0.024	0.121	0.039	0.145	0.026
min-var($\delta=0.95, \eta=4$)	0.138	0.053	0.137	0.055	0.149	0.055	0.137	0.081	0.166	0.067	0.010	0.020	0.112	0.071	0.150	0.059
min-var($\delta=0.95, \eta=10$)	0.130	0.101	0.129	0.105	0.149	0.129	0.146	0.170	0.200	0.168	0.003	0.018	0.099	0.137	0.146	0.141
min-var($\delta=0.85, \eta=1$)	0.134	0.025	0.134	0.026	0.137	0.022	0.131	0.031	0.140	0.026	0.061	0.031	0.126	0.037	0.138	0.023
min-var($\delta=0.85, \eta=2$)	0.136	0.048	0.136	0.049	0.142	0.046	0.132	0.060	0.149	0.054	0.036	0.038	0.126	0.074	0.143	0.048
min-var($\delta=0.85, \eta=4$)	0.134	0.084	0.134	0.087	0.144	0.094	0.133	0.111	0.160	0.109	0.019	0.047	0.129	0.140	0.147	0.098
min-var($\delta=0.85, \eta=10$)	0.123	0.138	0.127	0.147	0.147	0.200	0.135	0.208	0.175	0.229	0.012	0.075	0.140	0.251	0.141	0.188

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