# Estimated Interest Rates Term Structures and Implicit Inflation Methodology

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**Changes:** Inclusion of short-term references in estimating curves, footnote 3, page 6.

**SUMMARY**

## 1. INTEREST RATES TERM STRUCTURES

Building the Interest Rates Term Structures (ETTJ) for the different classes of securities is based on the same methodology as building the indicative range, available in the Index Dissemination System and in the ANBIMA's Secondary Market publication. The basic premise for estimating interest curves is that the price of a fixed-income bond is equal to the future cash flow promised by the issuer, brought to present value by a discount function[1].

$$P_{i,t} = \sum_{j=1}^{k_i} F_{i,j,t}\, b_t(T_{i,j}) + \varepsilon_{i,t}\ , \quad \forall\ i, t\,.$$

Where:

$F_{i,j,t}$: J-th payment (coupon and/or amortization) of the i-th bond at date t

$T_{i,j}$: term, in years (business days/252), in which payment j of the i-th bond occurs

$K_i$: number of payments of bond i

$P_{i,t}$: price of the i-th bond at date t

$\varepsilon_{i,t}$: error made by the model for bond i at date t

$b_t(T_{i,j})$: discrete discount function, defined by the equation:

$$\mathbf{b_t(T_{i,j}) = \frac{1}{\left(1 + r_t(T_{i,j})\right)^{T_{i,j}}}}$$

[1] Since we are dealing with bonds from the same issuer (the Brazilian federal government), the credit risk is the same for all bonds, therefore it is already compounded into the interest.

In the model proposed by Svensson (1994), the interest rate at date t for term $\tau$ , in years (based on business days/252), is expressed by the following equation:

$$r_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau}\right) + \beta_{3t}\left(\frac{1-e^{-\lambda_{1t}\tau}}{\lambda_{1t}\tau} - e^{-\lambda_{1t}\tau}\right) + \beta_{4t}\left(\frac{1-e^{-\lambda_{2t}\tau}}{\lambda_{2t}\tau} - e^{-\lambda_{2t}\tau}\right)$$

This model is widely used by several central banks, because it is a simple functional form that describes the entire time structure of interest rates with a small number of parameters. The equation format allows for a smooth and flexible structure that accommodates the various ETTJ formats observed in the data.

The factors of the structure have interpretation of level ($\beta_{1t}$), inclination ($\beta_{2t}$) and curvatures ($\beta_{3t}$ and $\beta_{4t}$). the parameters $\lambda_{1t}$ and $\lambda_{2t}$ characterize decay, determining where the loads of $\beta_{3t}$ and $\beta_{4t}$ reach their maximum.

Taking the limits of the equation above, we have the following:

$$\lim_{\tau \to 0} r_t(\tau) = \beta_{1t} + \beta_{2t}$$
$$\lim_{\tau \to \infty} r_t(\tau) = \beta_{1t}$$

Thus, $\beta_{1t}$ and $\beta_{2t}$ are, respectively, the long-term and short-term components of the ETTJ. On the other hand, $\beta_{3t}$ and $\beta_{4t}$ are the medium-term components, as the load that multiplies both, $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}$, starts at zero, is increasing at the beginning, and then tends toward zero when $\tau \to \infty$.

The ETTJ parameters ($\beta_{1t}$, $\beta_{2t}$, $\beta_{3t}$, $\beta_{4t}$, $\lambda_{1t}$ and $\lambda_{2t}$) are obtained by minimizing the sum of squared errors (square of the difference between the indicative price[2] and the resulting price of the model) of all bonds weighted by the inverse of the duration:

$$\text{Min} \quad \sum_{i=1}^{N} W_i \left(P_i - \sum_{j=1}^{k_i} F_{i,j}\, b_t (T_{i,j})\right)^2$$

As noted above, the objective function aims to minimize errors in the prices of the assets [3] (square of the difference between the indicative price[2] and the resulting price of the model). The procedure generates a homoscedastic result in terms of price errors, although it is heteroscedastic in this measure regarding the corresponding internal rates of return. This characteristic is a result of the difference in sensitivity to changes in rates due to changes in price for assets of different durations. With the aim of alleviating this problem, the Wi weight is used, since small differences in the prices of short-term assets imply large differences in their rates. The inverse of Duration was adopted because it more strongly penalizes the error for short-term assets, and nonetheless considers the influence of longer maturities.

This adjustment criterion was established considering the peculiarities of the domestic debt structure, such as the concentration of short-term maturities. The Association will monitor the profile of the Brazilian public debt to adjust the model in event there is a change in the maturity structure that justifies the change in the weight.

In constructing the indicative range, optimization is performed based on the four β parameters, since decay components $\lambda_{1t}$ and $\lambda_{2t}$ are maintained fixed. To allow a lower level of error and, consequently, a better adherence of the curve to the data, the ETTJ to be released will be obtained through the daily estimation of all six parameters of the model. Hence, the optimization becomes a little more complex, since it is highly non-linear, as shown by Bolder and Strélinski (1999).

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[2] Indicative price calculated based on the collection of the rates considered fair by the institutions for the bond, regardless of whether or not the paper has been traded.

[3] In order to include short-term references in estimating the curves, synthetic bonds with a maturity of 1 business day are adopted. In the case of Fixed ETTJ, the Estimate Selic Rate is adopted, which is calculated and published daily by ANBIMA on its website. In August 2021, a short-term reference was included for the ETTJ pegged to the IPCA (consumer inflation index) based on the ratio between the Estimated Selic Rate and ANBIMA's projection of the IPCA (or the official rate released by IBGE, when it is in force).

High non-linearity brings the risk that optimization may not reach a global minimum, but rather a local minimum, i.e., the risk of false convergence. This problem can be clearly observed when using traditional optimization methods (nonlinear least squares or maximum likelihood), since they are very sensitive to the initial parameters provided: the results obtained for the same date vary greatly, depending on the initial values. Additionally, estimation in this way leads to high volatility in the historical series of the parameters, an abundance of anomalous values, and a high frequency of structural changes, not justifiable by the evolution of the market for government bonds in Brazil.

Aimed at overcoming these problems, a genetic algorithm was developed in order to estimate the parameters of the Svensson model, enabling more satisfactory results, less volatile historical series, and a better fit to the data. For an even better result, the $\beta$ parameters found by the genetic algorithm are refined through traditional estimation, where the data obtained in the first one is used to define the initial region of the optimization.

The genetic algorithm, introduced by Holland (1975), is a search algorithm, inspired by evolutionary biology and applicable to different situations, the basic idea of which is to create a population of chromosomes that represent candidates for solving the problem. This population evolves over time through new generations. At each stage of evolution, the best individuals are selected and subjected to processes of crossover and mutation. These generations are created until the population converges to the optimal solution to the problem.

The efficiency of this algorithm, analogous to the idea of adaptation of individuals by evolving, lies in the ability to explore the accumulated information about an initially unknown search space, to bias subsequent searches toward more suitable spaces. Its development applied to the solution to the ETTJ estimation problem, according to the Svensson equation, was based on the work of Gimeno and Nave (2006) of the Central Bank of Spain.

## 2. BREAK-EVEN INFLATION IN THE TERM STRUCTURE

The reason for procedure for extracting the implicit inflation embedded in the interest rates term structure on the Brazilian market was the evolution of the management of federal bond debt since 2005, when the National Treasury concentrated its efforts on building a longer fixed-rate yield curve, starting with the issuance of NTN-F (fixed-rate bonds), and intensified the issuance of bonds pegged to price indexes, mainly NTN-B, indexed to the IPCA (consumer's inflation index).

In January of that year, the fixed-rate curve had vertices maturing in up to three years (NTN-F 1/1/2008). In December of the same year, the configuration of the term structure had vertices of up to six years (NTN-F 1/1/2012).

At the same time, the National Treasury promoted the placement of new maturities of IPCA-indexed bonds, with the aim of fixing vertices of the term structure, intensifying the volumes in primary placements. Between January and December 2005, the share of NTN-Bs in the debt rose from 3.3% to 7.7%. During 2006, these assets came to represent a major share of the liquidity of the secondary market for government bonds and, currently, Brazil is one of the countries with the highest number of outstanding inflation-linked bonds (***source:*** Barclay's World Gilb Index).

The new configuration of the structure of the classes of asset remuneration, liquidity and debt maturity allowed for the extraction of important information from the interest curves, such as the calculation of implicit inflation. Although there is noise in its interpretation as an estimate of future inflation, market participants use this information to calibrate asset pricing models based on the decomposition of risks associated with bonds and the way in which monetary policy is conduced.

## 2.1 Methodological Aspects of Implicit Inflation

Implicit inflation in interest curves is obtained based on a relationship known as Fisher's identity, which considers that the nominal interest rate is a composition between the real interest rate and the expected inflation for the period:

$$(1+r)=(1+\rho)\times(1+\pi) \tag{1}$$

Where, r is the nominal interest rate, ρ is the real interest rate, and π is inflation

Reorganizing equation (1), one can see that inflation can be extracted from the relationship between nominal and real interest rates. This spread found, called the Break Even Inflation Rate (BEIR), is the difference in expected returns between fixed-rate assets and indexed assets, i.e., the rate that will equalize the profitability between these two types of assets.

$$\pi=[(1+r)\div(1+\rho)]-1 \tag{2}$$

Where, r is the nominal interest rate, ρ is the real interest rate, and π is inflation

This method, widely used in the financial sector, cannot be directly interpreted as expectations of future inflation. The application of the simple form of Fisher's identity does not consider several important aspects, which end up causing an over- or under-estimation of the implicit inflation rate. Among them, the following stand out the most: the inflation risk premium embedded in the rates on fixed-rate assets, the difference in asset liquidity, and the difference in the structure of payments between fixed-rate and index-linked bonds.

The inflation risk reflects the probability attributed by agents to the expectation for inflation until the maturity of the bond will be lower than the one actually registered. Generally, the rates of return on fixed-rate financial assets embed a spread to offset this risk, the magnitude of which will depend on the degree of uncertainty in relation to the changes in the prices.

Regarding liquidity, it is worth noting that fixed-rate assets record a considerably greater business volume than IPCA-linked assets. For the latter, this difference translates into a higher premium demanded by investors, due to the potential difficulty in disposing of positions at the desired speed and price.

The use of zero-coupon curves in the calculation of the implicit inflation rate eliminates the problems generated by the difference in the payment structure of these two types of assets. Moreover, it allows for an analysis of the historical series of the BEIR, for selected fixed terms.

In terms of implicit inflation, the inflation risk embedded in fixed rates will generally put pressure on the BEIR, keeping it above the real expectations. Likewise, considering the lower liquidity of indexed assets, the tendency is for pressure to occur in the opposite direction. Therefore, we can conclude that the break-even inflation rate consists of the expected inflation and a spread reflecting the combination of risks embedded in the prices of the assets. Fisher’s identity can then be rewritten, considering the effect of this spread.

$$\pi = [(1+r) \div (1+\rho)] - 1 - Spread$$

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