# Credit curves

## Methodology

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ANBIMA

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# Credit Curves – Method

The increased supply and demand for credit assets after 2005 (mainly corporate bonds) and the Brazilian government's current policies to encourage long-term private financing by granting tax incentives and reducing requirements for issuing corporate debt, have served as an alert regarding the importance of developing mechanisms that support portfolio management and risk analysis, and increase market transparency.

Continuing the already consolidated work of daily disclosure of benchmark indicative prices for marking fixed-income portfolios and confirming its goal of providing information to the market, a methodology was developed to build private credit curves that reflect average risk spreads for different rating levels. Jointly with consultant and professor Caio Ibsen Rodrigues de Almeida of Fundação Getúlio Vargas, a parametric model was applied to the indicative prices of corporate bonds remunerated in DI (interbank deposit rate) and IPCA (consumer inflation index) that belong to ANBIMA's daily pricing sample.

The modeling of interest curves proposed by Nelson & Siegel synthesizes three movements indicated by researchers as the most common in these structures: level, slope, and curvature. A more in-depth study of these components was conducted by Litterman & Scheinkman in the paper titled "Common Factors Affecting Bond Returns" (1991). Motivated by the search for functions capable of representing these movements, the authors proposed modeling interest rates by the equation:

$$r(\tau) = \beta_0 + \beta_1\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_2\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$

In addition to interpreting the movements, it is possible to perform analyses by term through parameters β0, β1 and β2. The constant β0 represents the level of long-term rates, since the limit of the function when time tends toward infinity is β0 itself. β1 multiplies an exponential term that tends toward zero when the term tends toward infinity, thus representing short-term rates. Therefore, one can state that the very short-term rate is determined by (β0 + β1).

Finally, β2 is interpreted as a medium-term period, as it multiplies an exponential U-shaped term. There is a trend toward zero when the term is low, increasing as the term grows and reversing the trend with asymptote at zero in longer maturities.

There is also a parameter λ, which governs the exponential rate decay rate $r(\tau)$, where higher values imply faster decay and consequently better adjustment for short-term assets, and lower values imply slower decay and better adjustment in longer terms.

Svensson (1994), upon analyzing term structures for Sweden from 1992 to 1994, proposed the inclusion of an additional curvature to the functional form of Nelson & Siegel (1987), aimed at increasing flexibility and improving the fit to the data. Almeida et al. (2009) states that the inclusion of this fourth factor is important in modeling emerging markets, which generally have more volatile rates.

Diebold, Li and Yue (2008), when estimating interest curves with global factors for different countries, empirically noticed that most of the time the curvature is estimated with low precision due to the lack of data on very short and very long maturities, often hindering the curve's fit to the data. So the authors proposed using the Nelson–Siegel model with only level and slope factors to deal with the problem. The function that determines the spot interest rate for term τ, in this case, is reduced to the format below:

$$r(\tau) = \beta_0 + \beta_1\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)$$

Following the basic premise that the price of a fixed-income asset is the present value of its payment flows discounted by a market interest rate, the modeled discount function will be as follows:

$DF_i = 1/((1+DI)*(1+S))$, in the case of corporate bonds remunerated at the DI rate

$DF_i = 1/((1+REAL)*(1+S))$, in the case of corporate bonds remunerated according to the IPCA

Where: S is the credit spread modeled according to $r(\tau) = \beta_0 + \beta_1\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)$

CDI the adjustment rate of BM&F's DI Future interpolated for the term.

REAL: ANBIMA's zero-coupon IPCA curve discounting the risk premium[1]

Consequently, the asset's price is determined by the equation:

$$P = \sum_{i=1}^{n} DFi \; x \; FCi$$

Where: DFi is the discount function modeled for term i

FCi is the payment flow value within term i

The credit curves are estimated by solving the system of equations below, where the DI curve and credit spread curves by rating will be jointly estimated. As shown below, the DI curve will follow the Svensson functional form, while the credit curve will follow the Nelson–Siegel model without the curvatures.

$$r_{\mathrm{di}}(\tau) = \beta_{\mathrm{di}} + \beta_1\left(\frac{1-e^{-\lambda_{\mathrm{di}}\tau}}{\lambda_{\mathrm{di}}\tau}\right) + \beta_2\left(\frac{1-e^{-\lambda_{\mathrm{di}}\tau}}{\lambda_{\mathrm{di}}\tau} - e^{-\lambda_{\mathrm{di}}\tau}\right) + \beta_3\left(\frac{1-e^{-\lambda 2_{\mathrm{di}}\tau}}{\lambda 2_{\mathrm{di}}\tau} - e^{-\lambda 2_{\mathrm{di}}\tau}\right)$$

$$r_{\mathrm{AAA}}(\tau) = \beta_{\mathrm{AAA}} + (\beta_{1crédito})\left(\frac{1-e^{-\lambda_{\mathrm{credito}}\tau}}{\lambda_{\mathrm{credito}}\tau}\right)$$

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$$r_{\mathrm{A}}(\tau) = \beta_{\mathrm{A}} + (\beta_{1Crédito})\left(\frac{1-e^{-\lambda_{\mathrm{credito}}\tau}}{\lambda_{\mathrm{credito}}\tau}\right)$$

[1] To reconcile the spreads of the DI- and IPCA-remunerated corporate bonds, it is necessary to discount the government risk premium from the NTN-B base curve. The premium calculation performed in this process consists of the difference between ANBIMA's zero-coupon fixed curve, available on the Association's website, and B3's DI Futuro adjustment curve interpolated.

Where: $\tau$ is the term in years

βdi, βAAA, βAA, βA, β1, β1crédito, β2, β3, λdi, λ2di and λcrédito are model parameters.

It should be noted that the slope (β1Crédito) and decay (λ) parameters are common to the three spread curves, but each one has its own level parameter, representing the credit difference.

To estimate the parameters of the foregoing methodology, the weighted least squares technique is used. The weight is necessary because small discount-rate shocks influence long-term asset prices more significantly than short-term ones.

In this way, to determine the structures, the optimization problem below is solved:

$$\text{Min} \sum_{Rating/CDI} \sum_{i=1}^{n} (\text{W}_\text{i} (\text{P}_{\text{Modelo}} - \text{P}_{\text{Anbima}}) / \text{P}_{\text{Anbima}}))^2$$

Where, $W_i = \frac{1}{Duration_{\ (em\ anos)}}$

$P_{model}$: sum of the cash flow of the asset discounted to present value

$P_{Anbima}$: Price observed on the market

## Ratings

Some problems can arise when estimating curves by credit levels. First of all, there is no standardization in the nomenclature of notes issued by different agencies, or there may be notes at different levels for the same asset. Therefore, a method capable of generating a single classification for modeling is required.

The methodology used to select the credit risk associated with a given asset follows a process defined by the ANBIMA Asset Pricing Committee, in which “AAA,” “AA” and “A” ratings are assigned to each series participating in the sample.

# Database Processing

## 3.1 - Embedded Options

Some corporate bonds have options embedded in their clauses, which makes the risk of these bonds slightly higher, as the issuer has the right to redeem them before maturity at their par value[2]. For this class of assets, the investor requires a higher return to assume the risk, forcing a drop in price vis-à-vis those with the same rating without early repurchase clauses. Due to this price divergence, all corporate bonds that have early redemption or early amortization clauses in their indentures were eliminated from the sample.

## 3.2 - Short Term Assets and One-Day Synthetic Bond

Empirically, a significant reduction in asset liquidity is perceived on the market when it approaches maturity, in many cases leading to price distortions, which could erroneously influence the curves.

With the aim of minimizing this problem, but keeping some reference to short-term spreads, assets maturing within 21 business days were excluded from the sample, and a synthetic bond with a one-day maturity was inserted for each of the risk levels. Their prices are based on the 126 business day history of the very short-term rate, obtained by the sum of the level and slope parameters (B0 + B1).

## 3.3 - Identifying Outliers

As this is a sample of prices with heterogeneous issuers, it is common to find distorted observations capable of influencing the final results of the modeling. To deal with the problem, two combined statistical filters are applied:

a. Filter 1 (included after August 2016):

In order to identify possible observations with spread levels that are very far away from the behavior of their adopted risk rating, an initial filter based on the sample's interquartile range is applied.

To establish the filter limits, it is necessary to work with the first and third quartiles, according to the pre-established formulas listed below:

$L1 = Q1 - 3 \times (Q3 - Q1)$

[2] An asset is said to be at par when its price is exactly equal to its face value, i.e., with no premium or discount.

L2 = Q3 + 3 x (Q3 - Q1)

Where:

L1 – Lower limit/L2 – Upper limit;

Q1 – First quartile of total sample/Q3 – Third quartile of total sample.

b. Filter 2:

After disregarding the observations identified in filter 1 above, a metric based on Cook's residue analysis (1977) is applied, whereby the ratio between the mean square error including all assets and the mean square error without the i-th asset of the sample is calculated, according to the description below:

1. The mean square error is calculated with all assets in the sample (MSE).
2. One of the assets, asset i, is eliminated from the sample.
3. The curves are estimated again and the mean square error (MSEi) is calculated.
4. The MSE/MSEi ratio for asset i is found, here called critical value.

Procedures (2) to (4) are repeated until all assets have been selected. In this way, a vector of critical values is obtained for the entire sample, in such a way that the price of asset i will be far from the distribution whenever its critical value is too high. Finally, values that exceed the mean plus two standard deviations of the critical value vector are eliminated from the sample.

Moreover, in order to reduce the possible volatility caused by successive inflows and outflows of assets in periods of price adjustment, any observation eliminated in the proposed identification method will only return to the sample 21 days later.

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